EFFECTIVE PLACEMENT AND TUNING OF MULTIPLE TUNED MASS DAMPERS FOR MITIGATION OF VIBRATIONS IN STRUCTURES

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by

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Dedicated to my wife and my parents

for their endless love, support, and encouragement.

CERTIFICATE

This is to certify that the thesis titled, "**EFFECTIVE PLACEMENT AND TUNING OF MULTIPLE TUNED MASS DAMPERS FOR MITIGATION OF VIBRATIONS IN STRUCTURES**" being submitted by **Mr. Said Elias** to the Indian Institute of Technology (IIT) Delhi for the award of the degree of **DOCTOR OF PHILOSOPHY** is a record of the bonafide research work carried out by him. He worked under my supervision for the submission of thesis, which to my knowledge has reached the requisite standard as demonstrated by excellent international publications in journals and conferences.

Further, the contents of his research work, in full or in parts, have not been submitted to any other institute or university for the award of any degree or diploma to the best of my knowledge and belief.

New Delhi

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Said Elias

ABSTRACT

In the recent years, great attention has been given to research on and improve structural control devices, with particular focus on mitigation of wind and seismic response of structures. Tuned mass dampers (TMDs) are widely used in mitigation of wind and seismic response of the structures. In this study, the wind and seismic response control of the buildings, chimneys, and bridges installed with the TMDs are investigated. The modal frequencies and mode shapes of the buildings are determined for optimal placement of the TMDs. The TMD(s) is (are) placed where the mode shape amplitude of the buildings is largest or larger. The study is divided into three phases, wherein, the wind response control of tall buildings is focus of the first phase. In case of wind response control, a benchmark building is modeled with one lateral degree-of-freedom at each floor. The performance of single tuned mass damper (STMD) installed at the top floor of the benchmark building is compared with the STMD installed at different floors (locations) of the building. In each location, the STMD is tuned to the first few modal frequencies (subsequently). Placement, tuning frequencies, mass and damping ratios of the devices, are the parameters investigated to compare the effectiveness of these different STMD schemes. Afterwards, the multi-mode control strategy has been adopted; wherein, each TMD is placed where the mode shape amplitude of the building is the largest or larger in the particular mode being controlled and it is tuned with the corresponding modal frequency. The first five modes are controlled based on predetermined mass participation criterion. The performance of the distributed multiple tuned mass dampers (d-MTMDs) is compared with (i) STMD and (ii) all the multiple TMDs placed at top floor (MTMDs-all.top). Extensive parametric studies are conducted to compare the effectiveness of these three vibration response control schemes. The mass ratios, damping ratios of the devices, number of TMDs, and robustness of the TMDs are the parameters investigated. It is concluded that the placement of the STMD plays a significant role in improvement of the performance of the STMD, especially if it is tuned to the corresponding modal frequency where it is placed. In addition, it is concluded that the MTMDs-all.top exhibit improved performance as compared to the STMD to control the dynamic response. The use of the d-MTMDs is the most efficient alternative among the three, and they, being placed at various floors, are convenient to install requiring reduced space. Second phase of the present study is to investigate the performance of the STMD, MTMDs-all.top, and d-MTMDs in seismic response control of the buildings. In this case, first a benchmark building is controlled with the help of the

STMD installed at different floors (locations) of the building. In each location, a STMD is placed and tuned with different modal frequencies, while controlling first five modes. A comparison is also made between relative efficacy by adopting STMD and MTMDs-all.top, arbitrarily distributed multiple TMDs (ad-MTMDs), and proposed d-MTMDs. It is concluded that the d-MTMDs exhibit improved dynamic response control as compared to the STMD, MTMDs-all.top, and ad-MTMDs. It is possible that the structures will not remain in elastic range if subjected to the strong earthquake ground excitations. Therefore, the multi-mode vibration control of non-linear building with the d-MTMDs is investigated to assess its seismic vulnerability. The benchmark building is subjected to earthquakes such that the beams and columns are likely to undergo material non-linearity, which could possibly be controlled. The modal frequencies and mode shapes of the linear uncontrolled and controlled buildings are used in order to identify the optimal locations for the d-MTMDs. The seismic performance improvements by installing the d-MTMDs on the building are compared with the STMD, MTMDs-all.top, and ad-MTMDs. The variations of the inter-storey drift and residual inter-storey drift under different earthquakes are computed to study the effectiveness of using the STMD, MTMDs-all.top, ad-MTMDs, and d-MTMDs. Further, fragility functions for drift ratio are obtained to investigate the seismic vulnerability of the building without and with the control measures. It is concluded that the building generally remains within elastic range with installation of the MTMDs-all.top, ad-MTMDs, and d-MTMDs. Third phase of the present study includes investigations on the application of the proposed schemes for effective placement and tuning of the TMDs to the other structures such as reinforced concrete (RC) chimney and bridge. Effectiveness of the d-MTMDs for multi-mode control of chimney for along-wind and earthquake response is presented. It is observed that the d-MTMDs are more effective than the STMD and ad-MTMDs for the same total mass of the TMD/TMDs. The effectiveness of the d-MTMDs in seismic response control of bridges is also presented. Two horizontal components of earthquakes are simultaneously applied, while duly considering bi-directional interaction of the seismic response. Newmark’s step-by-step iteration method is used to solve the coupled differential equations of motion for the structure in all the studies mentioned above. It is concluded that the d-MTMDs installed as per the modal properties of the bridges help in effectively reducing the maximum pier base shear, support reaction, deck displacement, and deck acceleration in both the longitudinal and transverse directions.

सार

हाल के वर्षों में, संरचनात्मक नियंत्रण उपकरणों पर शोध और सुधार करने के लिए बहुत ध्यान दिया गया है, विशेष रूप से हवा की कमी और संरचनाओं के भूकंपी प्रतिक्रिया पर ध्यान केंद्रित किया गया है। ट्यूनेड द्रव्यमान डंपर्स (टीएमडी) व्यापक रूप से हवाओं के शमन और संरचनाओं के भूकंपी प्रतिक्रिया में उपयोग किया जाता है। इस अध्ययन में, टीएमडी के साथ स्थापित इमारतों, चिमनी और पुलों के हवा और भूकंपीय प्रतिक्रिया नियंत्रण की जांच की जाती है। इमारतों के मोडल आवृत्तियों और मोड आकृतियों को टीएमडी के अनुकूलतम स्थान के लिए निर्धारित किया जाता है। टीएमडी (एस) को रखा गया है (जहां) इमारतों की मोड आकार आयाम सबसे बड़ा या बड़ा है। अध्ययन को तीन चरणों में विभाजित किया गया है, जिसमें, उच्च इमारतों का पवन प्रतिक्रिया नियंत्रण पहले चरण का ध्यान केंद्रित है। वायु प्रतिक्रिया नियंत्रण के मामले में, एक बेंचमार्क इमारत प्रत्येक मंजिल पर एक पार्श्व डिग्री स्वतंत्रता के साथ तैयार की जाती है। बेंचमार्क भवन की ऊपरी मंजिल पर स्थापित एकल ट्यूनेड मास डैपर (एसटीएमडी) का प्रदर्शन एसटीएमडी के साथ तुलना में इमारत के विभिन्न मंजिलों (स्थानों) पर स्थापित किया गया है। प्रत्येक स्थान में, एसटीएमडी पहले कुछ मॉडल आवृत्तियों (बाद में) के लिए ट्यून किया जाता है। इन विभिन्न एसटीएमडी योजनाओं की प्रभावशीलता की तुलना करने के लिए नियुक्ति, ट्यूनिंग फ्रीक्वेंसी, मास और भिगोना अनुपात इसके बाद, बहु-मोड नियंत्रण रणनीति अपनाई गई है; जिसमें प्रत्येक टीएमडी रखा गया है, जहां भवन के आकार का आकार आयाम सबसे बड़ा या बड़ा है, विशेष मोड में नियंत्रित किया जाता है और यह इसी मोडल आवृत्ति के साथ ट्यून होता है पहले पांच मोड पूर्वनिर्धारित सामूहिक भागीदारी मानदंड के आधार पर नियंत्रित किए जाते हैं। वितरित कई ट्यूनेड मास डैम्पर्स (डी-एमटीएमडी) के प्रदर्शन की तुलना (i) एसटीएमडी और (ii) सभी मंजिल टीएमडी शीर्ष मंजिल (एमटीएमडी-ओल . टॉप) में की गई है। इन तीन कंपन प्रतिक्रिया नियंत्रण योजनाओं की प्रभावशीलता की तुलना करने के लिए विस्तृत पैरामीट्रिक अध्ययन किया जाता है। सामूहिक अनुपात, उपकरणों के अनुपात में भिगोना, टीएमडी की संख्या, और टीएमडी की मजबूती की जांच की जाने वाली मापदंड हैं। यह निष्कर्ष निकाला है कि एसटीएमडी का स्थान एसटीएमडी के प्रदर्शन के सुधार में महत्वपूर्ण भूमिका निभाता है, खासकर अगर इसे इसी मोडल आवृत्ति में रखा गया है जहां इसे रखा गया है। इसके अलावा, यह निष्कर्ष निकाला है कि एमटीएमडीएस-ओलटॉप प्रदर्शन में गतिशील प्रतिक्रिया को नियंत्रित करने के लिए एसटीएमडी की तुलना में प्रदर्शन में सुधार हुआ है। डी-एमटीएमडी का उपयोग तीनों में सबसे अधिक प्रभावी विकल्प है, और वे विभिन्न मंजिलों पर रखा जा रहा है, कम जगह की आवश्यकता को स्थापित करने के लिए सुविधाजनक हैं। वर्तमान अध्ययन का दूसरा चरण भवनों के भूकंपी प्रतिक्रिया नियंत्रण में एसटीएमडी, एमटीएमडी-ओल। टेस्ट और डी-एमटीएमडी के प्रदर्शन की जांच करना है। इस मामले में, पहले एक बेंचमार्क बिल्डिंग, इमारत के अलग-अलग मंजिलों (स्थानों) पर स्थापित एसटीएमडी की मदद से नियंत्रित किया जाता है। प्रत्येक स्थान में,

एक एसटीएमडी अलग मोडल आवृत्तियों के साथ रखा जाता है और पहले पांच मोड को नियंत्रित करते हुए। एसटीएमडी और एमटीएमडी-ओल। टोटल, मनमाने ढंग से वितरित किए गए कई टीएमडी (विज्ञापन-एमटीएमडी), और प्रस्तावित डी-एमटीएमडी को अपनाने के द्वारा तुलनात्मक प्रभावकारिता के बीच एक तुलना भी की जाती है। यह निष्कर्ष निकाला है कि डीटीएमडीडीएस एसटीएमडी, एमटीएमडीएस-ओल .टॉप और एड-एमटीएमडीएस की तुलना में गतिशील प्रतिक्रिया नियंत्रण में सुधार दर्शाता है। यह संभव है कि संरचनाएं इलास्टिक श्रेणी में नहीं रहेंगी अगर मजबूत भूकंप का मैदान उत्तेजनाओं के अधीन होता है। इसलिए, डी-एमटीएमडी के साथ गैर-रैखिक इमारत के बहु-मोड कंपन नियंत्रण की जांच उसके भूकंपीय भेद्यता का आकलन करने के लिए की जाती है। बेंचमार्क भवन भूकंपों के अधीन होता है, जैसे कि बीम और स्तंभों की सामग्री गैर-रैखिकता से गुज़रने की संभावना होती है, जिसे संभवतः नियंत्रित किया जा सकता है। डी-एमटीएमडी के लिए इष्टतम स्थानों की पहचान करने के लिए रैखिक अनियंत्रित और नियंत्रित भवनों के मोडल आवृत्तियों और मोड आकार का उपयोग किया जाता है। इमारत पर डी-एमटीएमडी स्थापित करके भूकंपीय प्रदर्शन में सुधार एसटीएमडी, एमटीएमडीएस-ओलॉट और विज्ञापन एमटीएमडीएस के साथ तुलना की जाती है। विभिन्न भूकंपों के अंतर्गत अंतर-मंजिली बहाव और अवशिष्ट अंतर-मंजिला बहाव के भिन्नरूपों को एसटीएमडी, एमटीएमडीएस-ओलटॉप, विज्ञापन-एमटीएमडी, और डी-एमटीएमडीएस के प्रयोग की प्रभावशीलता का अध्ययन करने के लिए गणना की जाती है। इसके अलावा, बहाव अनुपात के लिए विखंडन कार्यों को नियंत्रण उपायों के बिना और बिना इमारत के भूकंपी भेद्यता की जांच के लिए प्राप्त किया जाता है। यह निष्कर्ष निकाला है कि यह इमारत आमतौर पर एमटीएमडी-ओलटॉप, विज्ञापन-एमटीएमडी, और डी-एमटीएमडीएस की स्थापना के साथ इलास्टिक श्रेणी में बनी हुई है। वर्तमान अध्ययन के तीसरे चरण में प्रभावी प्लेसमेंट के लिए प्रस्तावित योजनाओं और टीएमडी के ट्यूनिंग को अन्य संरचनाओं जैसे प्रबलित कंकरीट (आरसी) चिमनी और पुल के रूप में शामिल करने की जांच शामिल है। डी-एमटीएमडी की प्रभावशीलता, हवा और भूकंप प्रतिक्रिया के लिए चिमनी के बहु-मोड नियंत्रण के लिए प्रस्तुत की जाती है। ऐसा पाया जाता है कि टीएमडी / टीएमडी के समान कुल द्रव्यमान के लिए एसटीएमडी और विज्ञापन-एमटीएमडी से डी-एमटीएमडी अधिक प्रभावी हैं।. पुलों के भूकंपी प्रतिक्रिया नियंत्रण में डी-एमटीएमडी की प्रभावशीलता भी प्रस्तुत की जाती है। भूकंप के दो क्षैतिज घटकों को एक साथ लागू किया जाता है, जबकि भूकंपीय प्रतिक्रिया के द्वि-दिशात्मक बातचीत पर विचार करना। न्यूमार्क के कदम-दर-चरण चलने वाला तरीका ऊपर वर्णित सभी अध्ययनों में संरचना के लिए गति के युग्मित अंतर समीकरण को हल करने के लिए उपयोग किया जाता है। यह निष्कर्ष निकाला है कि पुल के मोडल गुणों के अनुसार डी-एमटीएमडी स्थापित की गई है, अधिकतम अनुदैर्ध्य और अनुप्रस्थ दिशाओं में अधिकतम घाट आधार कतरनी, समर्थन प्रतिक्रिया, डेक विस्थापन, और डेक त्वरण को कम करने में मदद करता है।

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

MATHEMATICAL NOTATIONS

$[M_s]$	Mass matrix
$[C_s]$	Damping matrix
$[K_s]$	Stiffness matrix
$\{X_s\}$	Vector of displacement
$\{\dot{X}_s\}$	Vector of velocity
$\{\ddot{X}_s\}$	Vector of acceleration
N	Degrees of freedom (DOF) for the building
n	Degrees of freedom (DOF) for the TMD(s)
$\{G_s\}$	Placement vector
M_i	Lumped mass of the main structure
C_i	Damping of the main structure
K_i	Stiffness of the main structure
m_i	Mass of the i^{th} damper
c_i	Damping of the i^{th} damper
k_i	Stiffness of the i^{th} damper
X_i	Displacement at i^{th} degree of freedom
$\dot{X}_i$	Velocity at i^{th} degree of freedom
$\ddot{X}_i$	Acceleration at i^{th} degree of freedom
x_i	Displacement of the i^{th} damper
$\dot{x}_i$	Velocity of the i^{th} damper
$\ddot{x}_i$	Acceleration of the i^{th} damper
Ω_i	Natural frequencies of the structure
ω_i	Frequencies of the dampers
ζ_d	Damping ratio of the damper(s)
f_i	Tuning frequency ratios
μ	Mass ratio
m_t	Total mass of the damper(s)
M_t	Total mass of the structure
J_{w1}	Normalized RMS accelerations
J_{w2}	Normalized average percentage of the RMS acceleration

J_{w3}	Normalized RMS displacement
J_{w4}	Normalized average of the RMS displacement
J_{w7}	Normalized peak acceleration
J_{w8}	Normalized average of the peak acceleration
J_{w9}	Normalized peak displacement
J_{w10}	Normalized average of the peak displacement
J_{w11}	Normalized peak displacement of the damper
$\sigma_{\ddot{X}i}$	RMS accelerations of the i^{th} floor with control
$\sigma_{\ddot{X}75o}$	RMS acceleration of the 75^{th} floor without control
$\sigma_{\ddot{X}io}$	RMS acceleration of the i^{th} floor without control
σ_{Xi}	RMS displacements of the i^{th} floor with control
σ_{X76o}	RMS displacement of the 76^{th} floor
σ_{Xio}	RMS displacements of the i^{th} floor without control
$\ddot{X}_{pi}$	Peak acceleration of i^{th} floor with control
$\ddot{X}_{pio}$	Peak acceleration of i^{th} floor without control
X_{pi}	Peak displacement of i^{th} floor with control
X_{pio}	Peak displacement of i^{th} floor with control
$\ddot{X}_{p75o}$	Peak acceleration of 75^{th} floor without control
X_{p76o}	Peak displacement of 76^{th} floor without control
$\{F_t\}$	Wind load vector
$\{\phi_{i,j}\}$	Mode shapes
$[M_r]$	Modal mass participation
$[M_N]$	Mass matrix of the structure
$[C_N]$	Damping matrix of the structure
$[K_N]$	Stiffness matrix of the structure
$[m_n]$	Mass matrix of the dampers
$[C_n]$	Damping matrix of the dampers
$[K_n]$	Stiffness matrix of the dampers

$\ddot{X}_g$	Earthquake ground acceleration
g	Acceleration due to Earth's gravity
$\{r\}$	Vector of influence coefficients
$\lvert X_i(t)\rvert$	Absolute value of the controlled displacement of the i^{th} floor
X^{max}	Maximum uncontrolled displacement
$\lvert d_i(t)\rvert$	Absolute value of the inter-storey drift of the i^{th} floor
h_i	Height of each of the associated stories
d_n^{max}	Maximum uncontrolled inter-storey drift ratio
$\lvert \ddot{X}_i(t)\rvert$	Absolute value of the controlled acceleration of the i^{th} floor
$\ddot{X}^{max}$	Maximum uncontrolled acceleration
F_b^{max}	Maximum uncontrolled base shear
$\lVert X_i(t)\rVert$	Normed controlled displacement
$\lVert d_i(t)\rVert$	Normed controlled drift
$\lVert \ddot{X}_i(t)\rVert$	Normed controlled acceleration
t_f	Sufficiently large time to allow the response of the structure to attenuate to less than 0.1% of its maximum
$\lVert X^{max}\rVert$	Maximum normed uncontrolled displacement
$\lVert d_n^{max}\rVert$	Maximum normed uncontrolled drift
$\lVert \ddot{X}^{max}\rVert$	Maximum normed uncontrolled acceleration
$\lVert F_b^{max}\rVert$	Maximum normed uncontrolled base shear
J_{E1}	Normalized peak displacement
J_{E2}	Normalized peak drift
J_{E3}	Normalized peak acceleration
J_{E4}	Normalized peak base shear
J_{E5}	Normalized normed displacement
J_{E6}	Normalized normed drift
J_{E7}	Normalized normed acceleration

J_{E8}	Normalized normed base shear
J_{E10}	Normalized peak displacements of the damper
R_{w1}	Normalized peak displacement
R_{w2}	Normalized average peak displacement
R_{w3}	Normalized peak acceleration
R_{w4}	Normalized average peak acceleration
R_{w5}	Normalized RMS displacement
R_{w6}	Normalized normed average RMS displacement
R_{w7}	Normalized RMS acceleration
R_{w8}	Normalized average RMS acceleration
ζ_s	Structure damping ratio
D	Outer diameter
t_k	Thickness
E_c	Modulus of elasticity
Γ_i	Mass participation factors
R_{E1}	Percentage reduction in peak displacement
R_{E2}	Percentage reduction in peak acceleration
R_{E3}	Percentage reduction in peak base shear
R_{E4}	Percentage reduction in peak base moment
R_{E5}	Percentage reduction in RMS displacement
R_{E6}	Percentage reduction in RMS acceleration
R_{E7}	Percentage reduction in RMS base shear
R_{E8}	Percentage reduction in RMS base moment
R_{E9}	Shift in frequency
x_{20}	Controlled peak displacement at topmost node of the chimney
$\ddot{x}_{20}$	Controlled peak acceleration at topmost node of the chimney
f_b	Controlled peak base shear of the chimney
X_{20}	Uncontrolled peak displacement at topmost node of the chimney
$\ddot{X}_{20}$	Uncontrolled peak acceleration at topmost node of the chimney

F_{b}	Uncontrolled peak base shear of the chimney
σ_{20}	Controlled RMS displacement at topmost node of the chimney
$\ddot{\sigma}_{20}$	Controlled RMS acceleration at topmost node of the chimney
$\sigma_{f_{\mathrm{b}}}$	Controlled RMS base shear of the chimney
σ_{u20}	Uncontrolled RMS displacement at topmost node of the chimney
$\ddot{\sigma}_{\mathrm{u20}}$	Uncontrolled RMS acceleration at topmost node of the chimney
$\sigma_{F_{\mathrm{b}}}$	Uncontrolled RMS base shear of the chimney
Ω_{ci}	Controlled frequencies of the chimney
E_{d}	Damping energy
$J_{\mathrm{B1\text{-}x}}$	Reduction in pier base shear in longitudinal direction
$J_{\mathrm{B2\text{-}x}}$	Reduction of deck displacement in longitudinal direction
$J_{\mathrm{B3\text{-}x}}$	Reduction of deck acceleration in longitudinal direction
$J_{\mathrm{B1\text{-}y}}$	Reduction in pier base shear in transverse direction
$J_{\mathrm{B2\text{-}y}}$	Reduction of deck displacement in transverse direction
$J_{\mathrm{B3\text{-}y}}$	Reduction of deck acceleration in transverse direction
F_{p}	Controlled peak pier base shear
$\ddot{x}_{\mathrm{g}}$	Ground acceleration in longitudinal direction
x_{d}	Controlled peak displacement
$\ddot{x}_{\mathrm{d}}$	Controlled peak acceleration of deck
F_{po}	Uncontrolled peak pier base shear
x_{do}	Uncontrolled peak displacement
$\ddot{x}_{\mathrm{do}}$	Uncontrolled peak acceleration of deck
M^*	Modal mass
$\{Q\}$	Displacement vector
$\{\dot{Q}\}$	Velocity vector
$\{\ddot{Q}\}$	Acceleration vector
$\{\ddot{Q}_{\mathrm{g}}\}$	Bi-directional earthquake ground acceleration vector
$\ddot{y}_{\mathrm{g}}$	Ground acceleration in transverse direction
$\{Y_{\mathrm{i}}\}$	Displacement of the i^{th} node of the bridge in transverse direction
$k_{\mathrm{d\text{-}x}}$	Stiffness of the bridge in longitudinal direction

$k_{d\text{-}y}$	Stiffness of the bridge in transverse direction
$c_{d\text{-}x}$	Damping of the bridge in longitudinal direction
$c_{d\text{-}y}$	Damping of the bridge in transverse direction
$\bar{J}$	Mean
σ_J	Standard deviation
t	Integration time
Δt	Time steps
$\{x_s\}_t$	Displacement vector at time *t*
$\{\dot{x}_s\}_t$	Velocity vector at time *t*
$\{\ddot{x}_s\}_t$	Acceleration vector at time *t*
γ and β	Integration accuracy and stability parameters
a and *b*	Constants in integration time steps
K_E	Effective stiffness
ΔF_E	Effective force
Δx_s	Incremental displacement
$\Delta \dot{x}_s$	Incremental velocity
$\Delta \ddot{x}_s$	Incremental acceleration

ABBREVIATIONS

ad-MTMDs	Arbitrarily distributed multiple tuned mass dampers
APTMD	Adaptive passive tuned mass damper
ATMD	Active TMD
BTMDs	Bi-directional tuned mass dampers
CTMD	Coupled tuned mass damper
CMS	Component mode synthesis
CV	Coefficient of variation
DBE	Design basis earthquake
d-MTMDs	Distributed multiple tuned mass dampers
DOF	Degree of freedom
E-W	Transverse direction (East-West)
FDD	Friction damper device
FFT	Fast Fourier transform
FPLQG	Full passive linear quadratic Gaussian
JMA	Japanese Meteorological Agency
LQG	Linear quadratic Gaussian
MC	Monte Carlo
MCE	Maximum considered earthquake
MDOF	Multi-degree of freedom
MRFs	Moment resisting frames
MTMDs	Multiple tuned mass dampers
NC	Uncontrolled (no-control)
N-S	Longitudinal direction (North-South)
PC	Performance criteria
PD	Passive device
PGA	Peak ground acceleration
PSTMD	Pendulum stiffness tuned mass damper
RC	Reinforced concrete
RMS	Root mean square
RCFWBT	Reduced-order control frequency-weighted balanced truncation
RMBT	Reduced-order model balanced truncation
RP1LQG	Reduced passive 1 linear quadratic Gaussian

RP2LQG	Reduced passive 2 linear quadratic Gaussian
SAVFD	Semi-active variable friction dampers
SDOF	Single degree of freedom
SSI	Soil-structure-interaction
TMD	Tuned mass damper
TMDs	Tuned mass dampers
2D	Two-dimensional
3D	Three-dimensional