ION ACOUSTIC SOLITON PROPAGATION AND REFLECTION IN INHOMOGENEOUS PLASMAS : ANALYTICAL INVESTIGATIONS

by

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DEDICATED TO MY PARENTS

CERTIFICATE

This is to certify that the thesis entitled "ION ACOUSTIC SOLITON PROPAGATION AND REFLECTION IN INHOMOGENEOUS PLASMAS : ANALYTICAL INVESTIGATIONS" being submitted by Sanjay Singh, for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology (IIT), Delhi, is a record of bonafide research work he has carried out under my supervision. The results in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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PREFACE

A number of theories have been developed in the past to study the propagation characteristics and reflection of an ion acoustic soliton in homogeneous and inhomogeneous plasmas. In most of these theories, the electrons are considered to be isothermal, and the ion temperature is not taken into account. However, in a plasma ions possess finite temperature which affects the behaviour of solitary waves. In a weakly inhomogeneous plasma, the evolution of an initial perturbation into an ion acoustic soliton is sensitive to the ratio of the ion to electron temperature T_i/T_e. Also, for a collisionless plasma with relativistic effects, the behaviour of an ion acoustic soliton can be predicted accurately when the effect of ion temperature as well as plasma density is taken into account.

In this thesis, analytical investigations have been made for the propagation and reflection of an ion acoustic soliton in non-relativistic and relativistic plasmas. For the non-relativistic case, the effect of finite ion temperature on the propagation of an ion acoustic soliton has been studied in a weakly inhomogeneous plasma. Based on the assumption that the scale length of the plasma inhomogeneity is larger than the soliton width, a reductive perturbation analysis has been carried out to solve the basic fluid equations which finally lead to the KdV equation for an ion acoustic soliton. This equation is then solved to obtain expressions for the peak soliton amplitude, peak soliton ion density, peak soliton potential and the soliton width. It is shown that the peak value of the soliton amplitude increases and the soliton width decreases as the ion temperature is raised.

The reflection of an ion acoustic soliton from density gradient existing in a finite ion temperature plasma has also been studied. For this purpose, coupled differential equations for the oppositely travelling ion acoustic waves have been obtained. Fourier transformation with respect to time is used to solve the coupled differential equation for the wave travelling in the negative x direction. this gives the amplitude of the reflected wave to the lowest order. It has been shown that the amplitude of the reflected wave decreases as the ion temperature is raised. Furthermore, it has been shown that the amplitude of the reflected wave increases as it moves into the region of increasing plasma density. The reflection coefficient for the wave has also been defined and its variation with the increasing ion temperature has been studied.

In the last section of this thesis, investigations

have been made on the propagation characteristic of an ion acoustic soliton in a collisionless relativistic plasma having finite temperature ions. The ions are assumed to be streaming with high velocity in one dimension. As usual, reductive perturbation analysis has been used to obtain the KdV equation for the soliton, and expressions for the soliton phase velocity, soliton width, peak soliton amplitude, peak soliton ion density and peak soliton potential have been obtained. These results have been used to study the soliton behaviour in the plasma of the radiation belts surrounding the earth's atmosphere. The influence of the streaming velocity of the ions on the above mentioned soliton properties has also been investigated.

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