A THESIS ON

SECOND ORDER EFFECTS IN BOUNDARY LAYER PROBLEMS

By

S.R.N. Sastry
Department of Mathematics
Indian Institute of Technology
New Delhi

Submitted to the Indian Institute of Technology, New Delhi for the award of the Degree of Doctor of Philosophy in Mathematics

1967

A C K N O W L E D G E M E N T S

I am deeply indebted to Prof. M.K. Jain, D.Sc., Head of the Department of Mathematics, Indian Institute of Technology, Delhi, under whose supervision I have worked for the last two years, for his kind guidance, constant encouragement and help throughout this work.

My thanks are due to the authorities of the Indian Institute of Technology, Delhi for having placed at my disposal all the facilities necessary for my research and the authorities of the I.A.R.S., Delhi, who were kind enough to provide me with computation facilities on their computer IBM 1620.

I am under a great debt of gratitude to Principal P.R. Ramakrishnan, M.S. (M.I.T.) M.B.A. (New York), of Coimbatore Institute of Technology, Coimbatore, for his keen interest in my progress.

My thanks are also due to Dr. S.R.K. Iyengar for his kind help and to Mr. Dev Raj Joshi for his painstaking typing.

S R N Sastry

(S.R.N. Sastry)
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi.

CERTIFICATE

This is to certify that this thesis entitled "Second order effects in boundary layer problems", being submitted by Mr. S.R.N. Sastry to the Indian Institute of Technology, New Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him. Mr. S.R.N. Sastry has worked under my guidance and supervision for two years and has fulfilled the residential and other requirements for the submission of this thesis, which to my knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.

M.K. Jain

(M.K. Jain)
Head of the Department of Maths.,
Indian Institute of Technology,
Hauz Khas, New Delhi-29.

S Y N O P S I S

It has been recognised in recent times that an appropriate inviscid flow, together with boundary layers which adjust the slip velocity predicted by inviscid theory at solid boundaries, forms a uniformly valid leading term in the asymptotic expansion of the solution of the Navier-Stokes equations in ascending powers of viscosity for a wide class of problems, provided the solutions obtained do not break down. Attempts have been made to study higher order terms in the asymptotic expansion which take into account of, the second order effects related to longitudinal and transverse curvature, to vorticity of the main flow, to the displacement flow and further to the variation of stagnation enthalpy across the main flow if heat transfer at low Mach numbers is considered. The impact of the second order effects is increasingly felt due to their considerable influence on such important fluid phenomena as skin-friction, heat transfer, the velocity, vorticity and shear stress distribution.

The purpose of this thesis is to investigate the phenomena of flow and heat transfer in hydrodynamic and hydromagnetic flows, taking into consideration some of the second order effects. Attention is restricted to low speed flows and small temperature changes, so that the velocity field is that for an incompressible fluid, the temperature field being calculated subsequently. The second approximation is studied for laminar flow of a constant-

property fluid past analytic bodies free of separation. The thesis is divided into five chapters.

Chapter I consists of a description of the second order effects and a review of the investigations that has been carried out in the branch of higher approximations in boundary layer theory. A detailed analysis of the numerical method adopted for solving the constituent equations encountered in the subsequent chapters is also appended.

Certain aspects of the external vorticity problem in hydromagnetic flows is dealt in chapter II. The problem of the axi-symmetric steady flow of an incompressible viscous conducting fluid near a stagnation point, with vorticity in the oncoming flow varying linearly with the distance from the axis, is discussed in Part I. The magnetic field lines are circles parallel to the surface and the vortex lines form closed circles concentric to the axis of symmetry. For small values of the Alfven number and the vorticity interaction parameter, it is found that the presence of magnetic field diminishes the second order contribution due to external vorticity. Relative corrections for skin friction has been given and it is seen that the first order term for skin friction increases with the magnetic parameter. Part II deals with a similar problem in the two-dimensional stagnation point boundary layer, for a constant property fluid and a uniform imposed magnetic field. The main flow contains constant vorticity and both the induced magnetic field and charge density are assumed to be negligible. We find that the kinematic effect at the surface of the shear in the external stream increases with the magnetic parameter.

The first order contribution to skin friction is seen to decrease with the magnetic parameter. The first order viscous layer is found to be more sensitive to the presence of the magnetic field than the second order viscous boundary layer. An extension of the classical theory of simple shear flow to the hydromagnetic case in which a viscous electrically conducting, incompressible fluid flows past an electrically insulated semi-infinite flat plate in the presence of a uniform magnetic field parallel to the plate is made in Part III. A series solution for small values of the magnetic parameter is given, and the resulting ordinary differential equations have been numerically integrated.

To the first approximation, the effect of the magnetic field on first order boundary layer is seen to reduce the skin-friction. On the other hand it tends to increase the shear at the surface of that due to external vorticity, through interaction with the boundary layer. Expressions for relative corrections to the first and second order contributions have been given. First order viscous layer is found to be more affected than the second.

In chapter III is set out a study of laminar viscous and thermal boundary layers over cambered walls. This is essentially an extension of the Falkner-Skan family of self similar flows for longitudinal curvatures. The flow and energy equations have been derived for three cases of an assumed main flow varying as the powers of a longitudinal coordinate. Suitable curvature distributions have been chosen so as to give affine velocity profiles. The results are found to be essentially of the same character in all the three cases. It is seen that convex curvature decreases skin-friction and also heat transfer. The converse holds for concave curvatures. The curvature effect on heat transfer across a wall with constant temperature conducting heat to or extracting heat ~~from the main~~ from the main flow, is much smaller than the effect on skin friction. Numerical solutions of the constitutive ordinary differential equations have been given for various values of the curvature parameter.

The impact of distributions of normal suction at a surface on second order effects is the subject of study in chapter IV. The effects that have been included in the analysis are that of curvature, vorticity of the main flow, displacement flow and the variation of stagnation enthalpy across the main flow. The analysis is on the lines of the singular perturbation theory in which higher approximations in boundary layer theory are derived using the method of inner and outer expansions. We have chosen for investigation the three cases whose first order solutions are associated with the names of Hiemenz, Homann and Blasius. Suitable distributions of suction have been assumed and the resulting ordinary differential equations have been solved numerically for various values of the suction parameter. In general, the second order contributions to both skin friction and heat transfer, arising from the various secondary effects, are seen to increase with the suction parameter. The effect of suction is felt more in the viscous layer than the thermal layer to second order. The corrections are seen to preserve their signs independent of α, while their magnitudes increase with α, the suction parameter.

An attempt has been made in chapter V to discuss the vorticity interaction problem in time-dependent laminar boundary

C O N T E N T S

layers. A main flow of the form $\{ \frac{ax}{1 - \alpha at} - \omega y \}$, near a stagnation point is discussed for effects at the surface of the (constant ω) shear in the external stream; α denoting a parameter of unsteadiness. It is found that the surface shear, due to interaction with external vorticity of the boundary layer, decreases monotonically for positive values of α, the converse holding for negative values. A similar result is obtained in the neighbourhood of an axi-symmetric stagnation point for an unsteady oncoming flow containing vorticity proportional to the radius. The last section deals with the boundary layer growth due to shear flow over a flat plate when the diffusion rate is less than that of convection. It is found that in such a situation, the skin friction increases in both the viscous layers of first and second order.