

STATE AUGMENTATION APPROACH
TO SMOOTHING PROBLEMS

by

K.K. BISWAS

Thesis submitted in partial fulfilment
of the requirements for the degree of

DOCTOR OF PHILOSOPHY
in
ELECTRICAL ENGINEERING

to

INDIAN INSTITUTE OF TECHNOLOGY, DELHI
NEW DELHI

1973

PREFACE

The thesis describes the results of investigations carried out by the author in the area of Estimation Theory over the period September 1970 to November 1972 at the Indian Institute of Technology, Delhi. The author would like to express his deep sense of gratitude to Professor A.K.Mahalanabis of Electrical Engineering Department for his active help all through the progress of this study. He is also indebted to Professor S.C. Dutta Roy, ex-Head of Electrical Engineering Department for his interest and encouragement. He would like to thank his co-researchers, particularly Mr. S.R. Atre and Mr. K.L.S.Sharma, for many helpful discussions. Thanks are also due to the authorities of the Indian Institute of Technology, Delhi and the Council of Scientific and Industrial Research for providing with financial assistance during the tenure of the research fellowship.

New Delhi,
April 18, 1973

K.K.Biswas

CONTENTS

SECTION	PAGE
PREFACE	
<u>Chapter 1</u>	
REVIEW AND FORMULATION OF SMOOTHING PROBLEMS	
1.1 Introduction	... 1
1.2 Description and Classification of Estimation Problems	... 3
1.3 Some Problems of Data Smoothing	... 14
1.4 Review of Linear Smoothing Algorithms	... 17
1.5 Review of Time Delay and Distributed Smoothing Algorithms	... 28
1.6 Review of Nonlinear Smoothing Algorithms	... 30
1.7 Scope of the Thesis	... 33
<u>Chapter 2</u>	
LINEAR FIXED LAG SMOOTHING	
2.1 Introduction	... 39
2.2 The Case of Discrete Time System with White Noise	... 40
2.3 The Case of Coloured Noise	... 48
2.4 The Case of Time Delay	... 51
2.5 The Continuous Time Case	... 54
2.6 Numerical Examples	... 60
<u>Chapter 3</u>	
LINEAR FIXED POINT SMOOTHING	
3.1 Introduction	... 64
3.2 The Case of Discrete Systems with Independent White Noises	... 65
3.3 The Case of Correlated Noises	... 70

SECTION	PAGE
3.4 The Case of Time Delayed System with Coloured Observation Noise	... 73
3.5 The Case of Continuous Time Systems (Discretization Procedure)	... 77
3.6 Direct Derivation of Continuous Time Forms	... 80
3.7 Numerical Example	... 84

Chapter 4

SOME ASPECTS OF THE PROPOSED SMOOTHING ALGORITHMS

4.1 Introduction	... 88
4.2 Computational Aspects of the Fixed Lag Algorithm	... 90
4.3 Computational Aspects of the Fixed Point Algorithm	... 97
4.4 Stability Aspects of the Proposed Fixed Lag Algorithm	... 101
4.5 An Alternate 'Dynamic' Fixed Lag Smoother	... 104
4.6 Computational Aspects of Various Stable Fixed Lag Smoothers	... 109
4.7 Sensitivity Analysis	... 118

Chapter 5

NONLINEAR SMOOTHING

5.1 Introduction	... 123
5.2 Problem Statements	... 125
5.3 First Order Discrete Time Fixed Point Algorithm	... 127
5.4 First Order Discrete Time Fixed Lag Algorithm	... 131
5.5 Second Order Discrete Time Algorithms	... 133
5.6 Second Order Continuous Time Algorithms	... 139
5.7 Numerical Example	... 142

SECTION

PAGE

Chapter 6

SMOOTHING FOR DISTRIBUTED PARAMETER SYSTEMS

6.1	Introduction	...	145
6.2	System Description	...	147
6.3	The Case of Fixed Point Smoothing	...	148
6.4	The Case of Fixed Lag Smoothing	...	156
6.5	Numerical Example	...	164

Chapter 7

SUMMARY AND SOME SUGGESTIONS FOR FURTHER WORK

7.1	A Brief Summary	...	166
7.2	Some Suggestions for Further Work	...	172

REFERENCES	...	174
------------	-----	-----