SOME CONTRIBUTIONS TO THE METHODS OF LINEAR PROGRAMMING

by

VED PRAKASH
Computer Centre

submitted

in fulfilment of the requirements of

the Degree of

DOCTOR OF PHILOSOPHY

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

DECEMBER 1977

CERTIFICATE

This is to certify that the thesis entitled " Some Contributions to the Methods of Linear Programming" by Mr. Ved Prakash is a record of work carried out under our supervision and has not been submitted elsewhere for a degree.

New Delhi

December,1977

M. IBRAMSHA
Assistant Professor
Department of
Electrical Engineering
I.I.T. DELHI

P.C.P. BHATT
Professor
Department of Electrical Engineering
I.I.T. DELHI

ACKNOWLEDGEMENTS

I wish to express my deep appreciation to my thesis supervisors Professor P.C.P. Bhatt and Dr.M.Ibramsha for their encouragement, guidance and perceptive judgement throughout the preparation of this thesis. But for their help this thesis would not have been in this form.

I would like to thank Dr. Suresh Chandra, Department of Mathematics, I.I.T. Delhi and Dr. S.K. Gupta Department of Mathematics, I.I.T. Kanpur for their constructive criticism and many helpful suggestions during the initial stages of this work. I would fail in my duty without acknowledging gratitude to Prof. S.M.Sinha, Department of Operational Research, University of Delhi for introducing and motivating me to the field of Operations Research.

Thanks are due to Prof. M.K. Jain, Head, Computer Centre and Prof. O.P. Bhutani, Head, Department of Mathematics, IIT Delhi, for providing necessary facilities, extending all help and providing congenial atmosphere for work.

Thanks are also due to my teachers in the Computer Centre, I.I.T. Delhi. It is to them I owe my knowledge of computers. The operating staff of the computer centre deserve a special word of thanks for their co-operation in computational work all along my stay. I will never forget the association of my friends Shyam, Naveen, Sharma and Reddy.

My family members have in no small way contributed towards the completion of this work. To pushpa, my wife, is reserved my deepest appreciation for her patience, forebearance and understanding.

I gratefully acknowledge the financial support provided by the Indian Institute of Technology, Delhi, for this work.

Neat and clean typing was done by Mr. K. Bhatia.

ABSTRACT

The objective of this thesis is to provide algorithms for linear programming problems with near minimal iterations. The thesis is spread over seven chapters. The first chapter is introductory. In this chapter the past work and the state of present development is reviewed. The last chapter discusses the results and provides conclusions. Remaining five chapters discuss five algorithms proposed by the author. These five chapters have been grouped in Part A and Part B containing pivotal (basic solutions only) and hybrid (basic and non-basic solutions) algorithms respectively.

PART A : PIVOTAL TECHNIQUES

Several algorithms of linear optimisation problems with feasible region as convex polytope, are based on what might be called "vertex following" methods. These algorithms are based on the identification of a special class of feasible solutions (called vertices) and determination of an adjacency relation among them. Such algorithms start with a vertex (basic solution)

and proceed along successive adjacent vertices, according to some specified rule, until an optimal vertex is reached (or until it is shown that no optimal vertex exists). First method of this class, called the Simplex Method, was given by George F. Dantzig in 1947 and since then a number of variants, like Lemke's dual-simplex method (1954), Beale's method of leading variables (1954), primal-dual method of Dantzig-Ford-Fulkerson (1956) and many others, have been developed. All these methods are basically pivotal methods. In this part of the thesis author provides two new algorithms which are pivotal in nature.

The algorithm presented in Chapter II can be used to find a primal basic feasible solution and is based on the idea of selecting the pivotal row as one which corresponds to the farthest unsatisfied constrint either globally or along some axis on positive side. In the first case the algorithm has dual property and in the latter it has the primal property inherent in it. Therefore, we have considered two versions of this algorithm. In both versions, at every iteration a surrogate constraint, formed by adding all the violated constraints, serves as infeasibility function. In the primal version a column which has maximum rate of decrease of this function, is chosen as the pivotal axis. The violated constraint with maximum intercept on the positive side of this axis is taken as the pivotal row. In the dual version, we consider that set of non-basic columns which, if chosen as axes, have inter-sections with the globally farthest violated constraint on their positive sides. From this set, the column which has maximum rate of decrease in the value of the infeasibility function is selected as the pivotal column. In case there is no such column, that column which gives minimum increase is chosen. After pivoting, a check is made for primal feasibility. If it is achieved or inconsistency is identified the algorithm is terminated otherwise the iteration is repeated.

Chapter III of the thesis is devoted to another algorithm which uses both primal and dual iterations depending upon either the size of the problem or the state of the current solution. Given any tableau, which is both primal and dual infeasible, we use only the dual iterations until a primal feasible, or dual feasible, or both primal and dual feasible solution is reached or it is found that the problem has no finite optimum. If the problem has no finite optimum or the state reached is both primal and dual feasible the algorithm is terminated. If the state reached is primal (dual) feasible, then primal (dual) iterations are applied until termination. The pivot selection rule for the primal iteration is same as that of the primal-simplex method except that no consideration is given to the sign of the right hand side constants which now can be negative also. Similarly, a dual iteration is same as that of the dual-simplex method except that no consideration is given to the sign of reduced cost factors.

Part F : HYBRID TECHNIQUES

The term hybrid is used for those algorithms which do not restrict to the basic (vertex) solutions only but consider other solutions as well. One of the features of these techniques is that we can start with any feasible solution and move through or on the boundary of the convex polytope to reach at the optimum. The idea of proceeding through a convex polyhedral set (polytope) rather than around it has no doubt occurred to many researchers in mathematical programming and examples of such methods are Brown and Koopman's method

(1953), R. Frisch's multiplex method (1957), Motzkin's relaxation method (1954), Tompkin's projection method (1955), Zoutendijk's method of feasible directions (1960), and others. This class of methods is good for particular type of problems, for example, when the convex polytope consists of a large number of vertices and make pivotal methods inefficient. The algorithms in this part proposed by the author are found to be efficient enough in the above sense and particularly when the number of constraints (m) is very large as compared to the number of variables (n).

The first algorithm of this group, discussed in Chapter IV, applies a heuristic, the idea of which is to satisfy as many constraints along some specified direction as possible. The heuristic helps in reducing the number of infeasibilities (not necessarily the total infeasibility) significantly. The simplex algorithm is combined with it to assure the convergence as the heuristic alone does not always converge to a feasible solution. The heuristic is applied until there is an appreciable reduction in the number of infeasibilities in each iteration. Thus, the simplex algorithm is used at a stage when it is best suited during the process.

Chapter V contains another algorithm of the type hybrid which is complete in itself. It has two phases. Through phase I it arrives at a feasible solution (basic or non-basic) while in Phase II it attains optimum. The idea behind the method is as follows. From a vertex (basic solution) instead of moving to an adjacent vertex, a step is taken towards the interior of the cone formed by the current vertex such that

a) in Phase I, the infeasibility function given by the surrogate constraint corresponding to the current vertex is strictly decreased

and b) in phase II, the objective function is strictly improved.

From this new point a search for another vertex is made such that there is an improvement in the value of the infeasibility function or the objective function as the case may be. Thus, a movement of this type from one vertex to another, while strictly improving the appropriate function, can be considered as one iteration. To start a phase of this algorithm, a basic solution is not necessary. The algorithm is expected to be good for problems with large m and small n.

Chapter VI describes a hybrid algorithm that automatically avoids zig-zag paths without searching for a vertex-point from a non-vertex point (as is done by the algorithm of chapter VI). A linear programming technique is deduced from a conjugate gradient non-linear programming technique. A series of non-linear problems with spherical objective function are used to approach a linear programming problem by a limiting process.

All the algorithms proposed here have been developed aiming at the solutions of those problems with no primal and dual feasible solutions readily available. These algorithms have been tested and a comparison has been made with some of the existing algorithms. The results and conclusions are given in the last chapter.

C O N T E N T S

PART B