SOME PROBLEMS ON SANDWICH CONICAL SHELL

by

ASHOKE KUMAR RATH

APPLIED MECHANICS DEPARTMENT

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CERTIFICATE

This is to certify that the thesis entitled "Some Problems on Sandwich Conical Shell" being submitted by Shri Ashoke Kumar Rath to the Indian Institute of Technology, Delhi, for the award of the Degree of DOCTOR OF PHILOSOPHY IN APPLIED MECHANICS, is record of bonafied research work carried out by him. He has worked under my guidance and has fulfilled the requirements for the submission of this Thesis, which has reached the requisite standard.

The results contained in this Thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

S. Sengupta

(S. SENGUPTA)
APPLIED MECHANICS DEPTT.
Indian Institute of Technology
NEW DELHI

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It was a long march.

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ABSTRACT

Governing equations for 3 layer sandwich shell of orthotropic constructions along with general natural boundary conditions are developed from Hamiltonian variational principle. Basic equations of sandwich shell of revolution contain terms of static, inertia and thermal loadings. The shell in orthogonal curvilinear curvilinear coordinate system consists of two hard facings and the soft layer of core. The hard facings are of different materials of constructions and thicknesses and the variations of face stresses over the thicknesses of facings are negligible. The core consists of an elastic, orthotropic continuum, whose load carying capacity in the plane of sandwich is negligible. It is assumed that perfect continuity exists at the interfaces of facings and core. Strain displacement relations (kinematic) are most general in nature. Strain energy expressions of the facings, core, external work done on the shell along with the dynamical contributions provide the total strain energy expressions. Translational inertia in the 3 coordinate directions along with rotatory inertia and 2nd order normal inertia due to weak core effect are included in the dynamic contributions. Cross coupled translational and rotatory inertia are neglected as usual.

Based on generalised Hamiltonian principle, associated variational equations, known as Euler equations are presented

for small deformation theory. The effect of an arbitrary temperature distribution is also included based on Duhamel-Neumann constitutive laws. The effects of transverse shear stress, transverse normal stress are also taken into account.

Here accuracy of the order of Reissner's refined theory, has been achieved by two terms assumptions for u and v, which include the shear effects of core. The 3 terms assumptions for the transverse displacement enables us to study a linear variation of normal strain and this introduces constitutive equations for the transverse shear and transverse normal stresses.

This provides the most generalised equations of sandwich shell of revolutions in orthogonal curvilinear coordinates. Suitable modifications are made to Euler equations, to be applied to static, dynamic and thermal problems for homogeneous and sandwich plates and shells of various geometrical configurations.

So far no work exists for sandwich shell vibration problems taking into account transverse normal strain effects. So from the generalised formulation of the problem, an application is made for free vibration of a frustrum of conical sandwich shell. Here the equations in the orthogonal cartesian coordinate system are derived from the generalised sandwich shell equations, in earlier chapters. Here conical sandwich shell consists of two isotropic facings of equal thicknesses and identical material properties and can take only face parallel stresses. Core is orthotropic taking transverse shear and transverse normal stresses only. For the unsymmetrical case of free vibration of seven mode theory, the equations reduce to seven second-order ordinary differential equations after some approximations. They are non-dimensionalised and solved with the Galerkin's variational principle for the natural frequency and associated mode shapes. The thermal and static loadings are not taken into account.

For the clamped conical sandwich frustrum undergoing both unsymmetrical and axisymmetrical vibrations in seven mode, results are presented. Adopting the QR transformation method, eigen values and eigenvectors for following cases are evaluated.

(i) Unsymmetrical free vibration with weak core, taking compressibility of core into account. (7 mode theory).

(ii) Unsymmetrical free vibration with strong core, taking core as incompressible. (5 mode theory).

(iii) Symmetrical vibration with weak core, taking compressibility of core into account. (5 mode theory).

(iv) Symmetrical vibration with strong core, taking core as incompressible. (3 mode theory).

Further an attempt was made to obtain a close form solution for axisymmetric loading of sandwich frustrum conical shell with incompressible core. Governing equations for the above case are derived from the generalised equations of sandwich shell of revolution. Euler equations, which are in the form of three 2nd order simultaneous differential equations, in the 3 displacement variables, have been uncoupled to a single 6th order linear equation in terms of a single displacement variable. To avoid the singularity of the solution near the apex, a coordinate transformation has been made. The solution of the 6th order differential equation, which does not contain any singularity, is found with the help of a power series suitable for this particular case. The recurrent relation for the constants of the series has been provided.

The following problems are solved applying the above technique

(i) Conical sandwich frustrum shell, clamped at both ends and rotating with a constant angular velocity.

(ii) Conical sandwich frustrum shell, clamped at both ends, subjected to thermal loading.

Finally the results are discussed in detail and scopes for further research in this field are explored.

CONTENTS