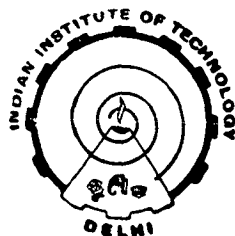


**STRESS ANALYSIS OF CRACK
IN PRESENCE OF RESIDUAL STRESS FIELD
USING THE FINITE ELEMENT METHOD**

by
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*A thesis submitted to the
Indian Institute of Technology, Delhi
for the award of the degree of
DOCTOR OF PHILOSOPHY*



**Department of Mechanical Engineering
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
1987**

TO MY PARENTS, BROTHERS AND SISTERS
WHOSE MORAL SUPPORT AND LOVE MADE THIS POSSIBLE

CERTIFICATE

This is to certify that the thesis entitled, "Stress Analysis of Crack in Presence of Residual Stress Field Using the Finite Element Method", being presented by Miss Geeta Agnihotri, is worthy of consideration for the award of the degree of 'Doctor of Philosophy' and is a record of the original bonafide research work carried out by her under my guidance and supervision and that the results contained in it have not been submitted in part or full to any University or Institute for the award of any degree or diploma.

I certify that she has pursued the prescribed course of research.

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ABSTRACT

A rectangular structural steel plate with a sharp crack has been selected to develop a two dimensional finite element model to obtain numerical results of displacements and stresses around the crack tip for linear elastic as well as elastic-plastic situation of loadings. Crack tip as well as cubic isoparametric elements are used throughout the analysis of elastic and elastic-plastic conditions. The spread of the plastic zone is shown to present the elastic-plastic boundaries for various load levels around the crack tip. Convergence tests are carried out to check the accuracy of the finite element model. The results of the convergence tests are compared with the theoretical Westergaard solution. Mouth Opening Displacements (MOD) are obtained experimentally under uniaxial stepped loadings for the same specimen size which is used in two and three dimensional finite element analysis.

A three dimensional finite element is developed to study the influence of residual stress on stress field of a single edge cracked plate subjected to an applied nominal stress of known magnitude. The nature and magnitude of residual stresses superposed on the applied nominal stress are idealized to present the residual stress state induced by the grinding process. The magnitudes of all the six components of stresses are estimated in presence of residual stress field near and away from the crack front. The variation of stresses around the crack front with the thickness of the cracked plate for different loading conditions is presented in the thesis.

It is found that the stress state around the crack front subjected to tensile loading is influenced by the presence of residual stress field.

ACKNOWLEDGEMENTS

The entire process of researching for this thesis has proved to be a many facet voyage for me. I could never have completed this doctoral work without the tremendous help and assistance which came to me from ever so many quarters.

Above all, I must express my indebtedness to Professor A. Mishra my guide and supervisor whose large-heartedness and unstinting, meticulous guidance at every conceivable stage leaves me with a deep sense of gratitude.

I am grateful to Dr. R.K. Pandey for his many suggestions and discussions I was able to conduct with him in the design of experimental work.

Professor B.L. Juneja and Professor V.R. Raghvan were also very helpful and I owe them my thankfulness for their constant affectionate encouragement.

This work was, of course initially possible because of the opportunity given to me for leave of absence from my college and the financial assistance provided by the Government of Madhya Pradesh and the Government of India under the QIP. I would particularly like to acknowledge my debt to Dr. B.L. Mehrotra, Principal, Maulana Azad College of Technology, Bhopal, for his encouragement all along.

It is not possible for me to individually thank ever so many friends and colleagues whose encouragement and warmth of heart enabled me to ultimately complete this work. I do, however, want them to know that I am deeply grateful to all of them.

I owe a word of special thanks to Shri V.P. Gulati who typed this thesis untiringly with exceptional diligence. Shri B.B. Arora and N.C. Saraswat also were of much help to me with the drawings.

Geeta Agnihotri
(Geeta Agnihotri)

Oct'87

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