

**STUDIES IN STATE AND PARAMETER ESTIMATION OF
DISTRIBUTED PARAMETER SYSTEMS**

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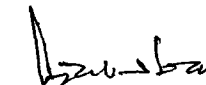
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CERTIFICATE

This is to certify that the thesis entitled, "Studies in State and Parameter Estimation of Distributed Parameter Systems", being submitted by Maya Shankar Prasad Sinha for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of the original bonafide research work carried out by him under our guidance and supervision. The results contained in this thesis have not been submitted to any other University or Institute for award of any degree or diploma.


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ABSTRACT

The thesis deals with the problem of state and parameter estimation in distributed parameter systems. Two different approaches to the problem of parameter estimation in distributed systems have been considered. One makes use of an equivalent state-variable lumped model whereas the other approach is more direct identification through the use of Walsh-functions. In the first method, both state and parameter estimation problems are involved which increase the dimension of the problem. Also, the method is restricted to linearized models. The solution is obtained through the use of a two-stage bootstrap state and parameter estimation algorithm. The second method of parameter estimation is more general in the sense that it is applicable to both linear and nonlinear, lumped and distributed systems. Properties of Walsh functions are used to establish suitable lemma that helps in getting equivalent linear algebraic equations from the original linear or nonlinear partial differential equations. Solutions to special cases like bilinear, time varying, time and space varying distributed parameter systems through the above approach have also been proposed. Some application of these methods to chemical plants, viz., packed bed and fluidized bed chemical processes, have been included by way of illustration.

In all five problems have been considered. In the first, the distributed system described by partial differential equations is

converted into coupled ordinary differential equations using finite-element and finite-difference methods. The resulting equations are then discretized. A two stage bootstrap identification algorithm has been employed to obtain the simultaneous estimates of states and parameters. The method overcomes the difficulties of higher dimensionality and nonlinearity involved in the use of extended Kalman filter for this purpose. A numerical example of a heat diffusion process has been solved for illustration.

The second problem considers parameter estimation of nonlinear lumped systems with fractional powers in nonlinearity. A solution using Walsh functions is proposed. The method is illustrated by a numerical example.

The problem of parameter-estimation in nonlinear distributed systems through the use of Walsh functions is considered next. A lemma based on the properties of Walsh functions is established which is crucial for the generalization of the identification procedure. Subsequently, the use of Walsh functions and the lemma has been extended to cover the identification of bilinear, time-varying, and time and space varying parameters of distributed systems.

Two case studies have been presented. In the first case, the problem of identification in a packed bed chemical process has been considered. The complicated integro-partial differential equation describing the process is first converted into a lumped state variable model using finite element method. The two-stage bootstrap

identification algorithm is then employed for estimation of states and parameters of the process. In the second case study, the problem of parameter estimation in a fluidized bed heat exchanger system has been considered. The process is described by coupled nonlinear partial differential equations. The identification of the process is achieved through the use of Walsh functions.

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