CERTIFICATE

This is to certify that the thesis entitled :

'NEW ANALYTIC SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS OF MATHEMATICAL PHYSICS VIA ISOVECTOR, EQUIVALENCE TRANSFORMATIONS AND DIRECT METHODS'

which is being submitted by Ms. Lipika Bhattacharya, Research Scholar, Mathematics Department to the Indian Institute of Technology, Delhi, for the award of the DEGREE OF DOCTOR OF PHILOSOPHY in MATHEMATICS, is a record of bonafide research work carried out by her under my guidance and supervision and has fulfilled all the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

The study being presented as a thesis entitled "New Analytical Solutions of Nonlinear Partial Differential Equations of Mathematical Physics via Isovector, Equivalence Transformations and Direct Methods" highlights the usefulness of the analytical methods of solution mentioned in the title by way of application to a large number of problems from the realm of theoretical physics. The isovector and the equivalence transformation approaches have direct links with the Lie's theory of continuous groups of transformations. Precisely speaking, the isovector fields and the equivalence generators, in the isovector approach and the equivalence transformations method respectively, play the same role as that of the infinitesimal operator in Lie's theory. Using the similarity transformations obtained from these one-parameter groups and which essentially reduce the number of independent variables by one, the nonlinear partial differential equations (nlpdes) in two independent variables are reduced to nonlinear ordinary differential equations (nlodes), the solutions of which eventually yield solutions of the nlpdes. Beside these two group theoretic methods, some other methods, which donot evoke group invariance, are also used. These direct methods are WTC technique or analysis of the weak Painlevé property for nlpdes, Clarkson and Kruskal's direct method and travelling wave solutions technique by Jeffrey and Xu. In the next few paragraphs we briefly mention the material presented in the seven chapters of the thesis.

Chapter I gives a brief description of the methods used in handling equations of the remaining chapters and the survey of some related works. In Chapter II, some

Klein-Gordon and Liouville type equations in n-Euclidean dimensions are taken up for similarity reduction. The resulting nlodes are solved for some special forms of the arbitrary functions involved and the results are tabulated.

A Poisson type equation in n-Euclidean dimensions in which the inhomogeneous part is an arbitrary function of the dependent and independent variables as well as the first order partial derivatives of the dependent variable, is studied in Chapter III by the equivalence transformation and isovector approaches and the results are compared. Isogroups are calculated for the Harry Dym equation in Chapter IV and ten group theoretic reductions are tabulated, some of which are solved to get some implicit and some explicit solutions.

In Part I of Chapter V, an integro-differential equation derived by Hirota and Satsuma by Bäcklund transformation of the Boussinesq equation is taken up and some exact solutions are reported. In the second part, the third order Boltzmann equation is studied via the isovector and WTC techniques, to get exact solutions. It is observed that some of the similarity transformations work for the $(p+1)$ th order equation as well, from which results for the Krook-Wu model $(p=1)$ are deduced.

In Chapter VI the small disturbance potential flow equation of nonequilibrium transonic gasdynamics is considered through the isovector approach and the direct method due to Clarkson and Kruskal, which again transforms the nlpde to nlode but does not require invariance under a group of transformation. Beside giving exact solutions of the equation under consideration, solvability and nilpotency of the Lie algebra of the transformation groups yielded by the isovectors are examined.

Unlike Chapters II - VI wherein we utilize group theoretic methods for determining the exact solutions of nlpdes, in this chapter, the technique given by Jeffrey and Xu to find travelling wave solutions, is applied to a class of conservative nonlinear evolution equations that embeds nonlinearity and dispersion. Results for some physically interesting particular cases such as the generalized Kuramoto-Sivashinski equation, the Korsunsky equation in a special case, a generalized Boussinesq equation, a modified Boussinesq equation, a nonlinear atomic chain model equation etc. are deduced from the general solution.

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