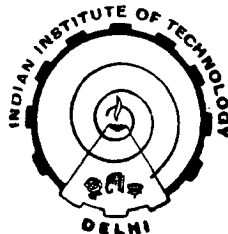


FORCED AND FREE CONVECTION WITH MASS TRANSFER IN NONHOMOGENEOUS ISOTROPIC POROUS MEDIA

By
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*A THESIS SUBMITTED TO THE
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FOR THE AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY*



Department of Mathematics
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February, 1987

CERTIFICATE

This is to certify that the thesis entitled "FORCED AND FREE CONVECTION WITH MASS TRANSFER IN NON-HOMOGENEOUS ISOTROPIC POROUS MEDIA" which is being submitted by Mr. Nandakumar K to the Indian Institute of Technology, Delhi, for the award of the degree of DOCTOR OF PHILOSOPHY in Mathematics, is a record of bonafide research work carried out by him under my guidance and supervision.

The thesis has reached the standard, fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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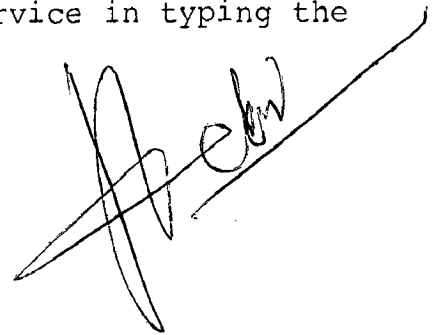
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Finally, I wish to express my thanks to Ms Neelam Dhody to her skillful service in typing the manuscript.

A handwritten signature in black ink, appearing to be 'Nandakumar.K', written over a diagonal line.

(NANDAKUMAR.K)

ABSTRACT

The work presented in this dissertation entitled "FORCED AND FREE CONVECTION WITH MASS TRANSFER IN NON-HOMOGENEOUS ISOTROPIC POROUS MEDIA", is divided into five chapters. An introduction to the problems considered is given in the first chapter with a brief account on flow through porous media, dispersion in porous media and the characterisation of nonhomogeneity.

Chapter II, provides a study of forced flow against dispersion in finite porous media assuming porosity variations in the flow direction. Steady and unsteady cases are considered with chemical reaction of the solute in the bulk of the fluid and adsorption of the solute on the solid surface. Analytical solutions are obtained for the steady state case and the unsteady problems are solved numerically using finite difference method. The effect of physical parameters, namely, ratio of longitudinal dispersivity to porous medium length, parameters of chemical reaction and adsorption are studied for different rates of porosity variation.

In Chapter III, numerical solutions are presented for the problem of flow against dispersion with permeability variations in the transversal direction of flow, assuming steady state conditions. The solute is assumed to undergo chemical reaction in the bulk of the fluid. Two types of boundary conditions are assumed for concentration namely, constant concentration conditions and no flux conditions. The effect of different physical parameters which are obtained by nondimensionalising the governing differential equations are analysed for different rates of porosity and particle size variations.

An analysis of the effect of channelling and flow maldistribution on mass transfer by convection and dispersion is made in the fourth chapter using Brinkman's model. The mass transfer is considered with the chemical reaction of the solute in the bulk of the fluid and heterogeneous reaction at the walls which are assumed to be catalytic. The effect of these processes on the concentration distribution is analysed with porosity and particle size variations by solving the differential equations using finite difference method.

The last chapter provides an extensive numerical study on the effect of nonhomogeneity resulting from particle

size variations on natural convection of Boussinesq fluids occurring in enclosed porous media, assuming steady state conditions. The problems considered are mass transfer by natural convection in a vertical porous annulus, mass and heat transfer by natural convection in a porous layer with different types of boundary conditions for the vertical walls and salt water encroachment in coastal aquifers. The flow distribution and heat and mass transfer rates were found to be significantly influenced by the non-homogeneity of the porous media.

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