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SOME ASPECTS
OF
THE POSTAGE STAMP PROBLEMS

By

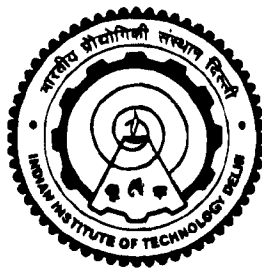
GEETA

Department of Mathematics

Submitted

*in fulfillment of the requirements
of the degree of Doctor of Philosophy*

to the

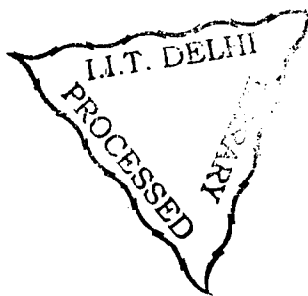


Indian Institute of Technology Delhi

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To My Husband Ajay

CERTIFICATE

This is to certify that the thesis entitled 'SOME ASPECTS OF THE POSTAGE STAMP PROBLEM' submitted by Geeta to the Department of Mathematics, Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy, is a record of bona fide research work carried out by her under my guidance and supervision. The thesis has reached the standards fulfilling the requirement of the regulations relating to the degree. The work contained in this thesis has not been submitted to any other university or institute for the award of any degree or diploma.



Dr. A. Tripathi

(Supervisor)

January 2009

Department of Mathematics

Indian Institute of Technology Delhi

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Geeta
Geeta

ABSTRACT

The Postage Stamp Problem (PSP) can be stated in terms of placing postage stamps on an envelope with limited space. Suppose we have an unlimited supply of postage stamps of denominations $a_1, a_2, \dots, a_k$, and we wish to affix them on an envelope with place for no more than h stamps. The ‘local’ version of the PSP asks for the smallest stamp value that cannot be affixed under the given limitation, while in the ‘global’ version we replace the given set of stamp denominations by arbitrary collections of k stamps and seek the largest of these. The PSP was apparently introduced by Hofmeister in 1936, and is known to be NP-hard.

Our approach differs subtly from the previous ones by considering the solution space of the representation of a positive integer as a linear combination of elements from a given basis. A section is also devoted to the visualization of the problem in better manner. This thesis improves the results given by [22], and also contributes many general results like knowledge of $n(h, A_k)$ which is of a tremendous aid in improving estimate of $n(h, k)$. We give bounds for a_i as a function of i, h, k to reach the

solution set of $n(h, k)$. Also, we provide some results which estimate the magnitude of a_{i+1} relative to a_i in the admissible and extremal basis. As an application of the visualization of the problem one part of the thesis deals in giving a solution to the $k=3$ case of the LPSP. This is an improvement to Selmer's work [21], which in turn provided solution to the global aspect with three stamp denominations, $n(h, 3)$, and the corresponding extremal basis case.

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