MIXED DUALITY AND MIXED SYMMETRIC DUALITY IN MATHEMATICAL PROGRAMMING

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Certificate

This is to certify that the thesis entitled *MIXED DUALITY AND MIXED SYMMETRIC DUALITY IN MATHEMATICAL PROGRAMMING* which is being submitted by *ABHA* for the award of the degree of *DOCTOR OF PHILOSOPHY* in *MATHEMATICS* to the *INDIAN INSTITUTE OF TECHNOLOGY, DELHI*, is a bonafide research work done under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

(SURESH CHANDRA)
Professor, Department of Mathematics,
Indian Institute of Technology, Delhi
Hauz Khas, New Delhi – 110 016
INDIA

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Abha

ABHA GOYAL

Abstract

In the thesis entitled *Mixed Duality And Mixed Symmetric Duality In Mathematical Programming*, the thrust is on the formulation and conceptualization of the concept of mixed duality and use the same to explain the construction of the Mond – Weir type duals in nonlinear programming. Thus this thesis is primarily devoted to the study of mixed duality and saddle point optimality criteria via mixed Lagrange function for a class of scalar and multiobjective programming problems. Further a new symmetric dual model called the mixed symmetric dual, is presented for the above mentioned class of problems which unifies the two well known existing symmetric dual models available in the literature. Some of these results are also extended to the situations where certain variables are constrained to be elements of some arbitrary sets e.g. the set of integers.

The concept of mixed duality seems to be interesting and useful, both from theoritical as well as from algorithemic point of view. Although this thesis does not address the computational advantages of the mixed dual formulation, it is hoped that the flexiblity of the choice of constraints to be put in the Lagrange function can be exploited to develop certain efficient solution procedures for solving nonlinear programming problems.

The thesis consists of six chapters. Chapter 1 is an introductory one and contains a brief review of the related work and summary of work presented in the thesis.

In Chapter 2 the main result observed is the fact that the Mond and Weir dual is connected to the mixed Lagrange function exactly in the same manner as the Wolfe dual is connected to the usual Lagrange function. Thus the results of this chapter provide a very general way of writing the mixed dual for a mathematical programming problem once a correct choice of the mixed Lagrange function has been made.

The purpose of Chapter 3 is to generalize and also to unify two existing symmetric duality models in the literature. For this, first the symmetric duality under F-convexity F-concavity / F-pseudoconvexity F-pseudoconcavity is studied and then these resulsts are used to investigate symmetric duality for various types of minimax mixed integer programming problems. With the aim of unifying the existing models, a symmetric dual formulation for a class of nonlinear programming problems is also presented in this chapter, which is then extended to study mixed symmteric duality in minimax mixed integer programming problems.

Chapter 4 studies the mixed duality and mixed symmetric duality in multiobjective programming. A critical study of some recent results on symmetric duality in multiobjective programming has been made and certain basic shortcomings have been noticed. Further a modified symmetric dual pair of multiobjective programs is constructed which takes care of these shortcomings and unifies the two existing models for such programming problems.

Chapter 5 is focussed on the study of duality for a class of nonlinear programming problems in which the objective as well as the constraint functions contain terms of the support function of certain compact convex sets. Further the nondifferentiable symmetric duality results available in the literature are extended to study the symmetric duality in fractional and minimax mixed integer programming problems.

The purpose of Chapter 6 is to introduce a pair of second order mixed symmetric dual nonlinear problems and extend the same to study the mixed second order symmetric duality in minimax mixed integer programming.

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