MODELLING AND ANALYSIS OF HIGHER ORDER MODES IN OPTICAL FIBERS & COMPONENTS

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by

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Submitted
in fulfilment of the requirements of the degree of
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To my Family

CERTIFICATE

This is to certify that the Thesis entitled **"MODELLING AND ANALYSIS OF HIGHER ORDER MODES IN OPTICAL FIBERS & COMPONENTS"** being submitted by **Mr. Ajay Kumar** to the Department of Physics, Indian Institute of Technology Delhi, for the award of the degree of '**Doctor of Philosophy**' is a record of Bonafide work carried out by him. He has worked under our supervision and guidance and has fulfilled the requirements for the submission of the thesis, which in our opinion has reached the requisite standard.

The results contained in this work have not been submitted, in part or fully, to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

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Ajay Kumar

ABSTRACT

Over the past two to three decades, optoelectronic devices have become increasingly important. Developing these devices requires effective modeling and analysis techniques, as analytical solutions are not feasible for non-uniform structures, making numerical methods essential. Optical fibers are a very important class of components used in optoelectronics and optical communication. Optical fibers are classified by modes: single-mode fibers (SMFs) carry only the fundamental mode, while multimode fibers (MMFs) support higher-order modes (HOMs) also. HOMs enhance communication capacity through mode-division multiplexing and enable advanced sensing.

In this thesis, for optical fiber and components, cylindrical coordinates are used to describe Maxwell's equations and wave equations. Propagation methods solve the wave equation, with various approaches developed for beam propagation in photonic structures. Scalar wave approximation offers efficient results in low index contrast structures, and solutions vary based on the specific parameters of problem. We have developed an efficient method for wave propagation in azimuthally symmetric fiber structures, supporting both paraxial and non-paraxial wave propagation. The method reduces the scalar wave equation to 2D from 3D which requires only fewer sample (collocation) points, offering significant computational efficiency and accuracy. This method is applicable to all higher order modes including OAM modes.

We have examined the propagation of higher order modes in fiber tapers and fiber Bragg gratings (FM-FBGs) for sensing applications. It uses the split-step non-paraxial method to compute reflections and spectral responses. Our study on a fiber sensor based on FM-FBG for simultaneous temperature and refractive index measurement using LP_{01} and LP_{11} modes shows a

good agreement with experiments. LP_{11} mode has higher refractive index sensitivity, while temperature sensitivity is similar for both. We have also examined the issue of cross-sensitivity between the two parameters in simultaneous measurement. We have concluded that cross-sensitivity is very small and largely unreliable to quantify. Sensor tip modification improves temperature sensitivity by 15.76% for LP_{01} and 18.59% for LP_{11}. The effects of ring-core fiber parameters on mode coupling, effective area, and differential mode delay have also been analyzed. Finally, we have proposed a stable optical tweezer probe using a ring-core fiber and graded-index multimode fiber, which supports stable OAM modes.

We have proposed a new method for obtaining mode and the modal loss in leaky guiding structure. The leaky mode is obtained as the limiting-guided mode of the given leaky structure. The loss is obtained by propagating this mode through the leaky structure. The mode computation and the propagation loss have been carried out using the collocation method. We have optimized a Bragg fiber with nine periodic bilayers of low and high refractive index to improved mode confinement. The structure keeps the mode well-confined for wavelengths from $0.6\ \mu m$ to $1.242\ \mu m$, with very low loss. Higher-order modes leak out and have been handled with the ABC boundary conditions.

Finally, for the modelling of the structures with unevenly distributed guiding regions, redistributing collocation points is necessary. This has been achieved with a transformation function that ensures a one-to-one correspondence between the transformed and real space. We have demonstrated improved accuracy in modal computation for large core Bragg fiber. We have also shown that the solid core Bragg fibers can also efficiently modelled using this variable transformed collocation method.

सार

पिछले दो से तीन दशकों में, ऑप्टोइलेक्ट्रॉनिक डिवाइस बहुत महत्वपूर्ण हो गए हैं। इन डिवाइस को विकसित करने के लिए प्रभावी मॉडलिंग और विश्लेषण तकनीकों की आवश्यकता होती है, क्योंकि गैर-समान संरचनाओं के लिए विश्लेषणात्मक समाधान संभव नहीं हैं, जिससे संख्यात्मक विधियाँ आवश्यक हो जाती हैं। ऑप्टिकल फाइबर ऑप्टोइलेक्ट्रॉनिक्स और ऑप्टिकल संचार में उपयोग किए जाने वाले घटकों का एक बहुत ही महत्वपूर्ण वर्ग है। ऑप्टिकल फाइबर को मोड के आधार पर वर्गीकृत किया जाता है: सिंगल-मोड फाइबर (SMF) केवल मूल मोड को ले जाते हैं, जबकि मल्टीमोड फाइबर (MMF) उच्च-क्रम मोड (HOM) का भी समर्थन करते हैं। HOM मोड-डिवीजन मल्टीप्लेक्सिंग के माध्यम से संचार क्षमता को बढ़ाते हैं और उन्नत संवेदन को सक्षम करते हैं।

इस थीसिस में, ऑप्टिकल फाइबर और घटकों के लिए, मैक्सवेल के समीकरणों और तरंग समीकरणों का वर्णन करने के लिए बेलनाकार निर्देशांक का उपयोग किया जाता है। प्रसार विधियाँ तरंग समीकरण को हल करती हैं, जिसमें फोटोनिक संरचनाओं में बीम प्रसार के लिए विभिन्न दृष्टिकोण विकसित किए गए हैं। स्केलर तरंग सन्निकटन कम सूचकांक विपरीत संरचनाओं में कुशल परिणाम प्रदान करता है, और समाधान समस्या के विशिष्ट मापदंडों के आधार पर भिन्न होते हैं। हमने एजिमुथली सममित फाइबर संरचनाओं में तरंग प्रसार के लिए एक कुशल विधि विकसित की है, जो पैराक्सियल और गैर-पैराक्सियल तरंग प्रसार दोनों का समर्थन करती है। यह विधि स्केलर तरंग समीकरण को 3D से 2D तक कम कर देती है जिसके लिए केवल कम नमूना (कोलोकेशन) बिंदुओं की आवश्यकता होती है, जो महत्वपूर्ण कम्प्यूटेशनल दक्षता और सटीकता प्रदान करता है। यह विधि OAM मोड सहित सभी उच्च क्रम मोड पर लागू होती है।

हमने संवेदन अनुप्रयोगों के लिए फाइबर टेपर और फाइबर ब्रैग ग्रेटिंग (FM-FBGs) में उच्च क्रम मोड के प्रसार की जांच की है। यह प्रतिबिंबों और वर्णक्रमीय प्रतिक्रियाओं की गणना करने के लिए विभाजित-चरण गैर-पैराक्सियल विधि का उपयोग करता है। LP_{01} और LP_{11} मोड का उपयोग करके एक साथ तापमान और अपवर्तक सूचकांक माप के लिए FM-FBG पर आधारित फाइबर सेंसर पर हमारा अध्ययन प्रयोगों के साथ एक अच्छा समझौता दिखाता है। LP_{11} मोड में उच्च अपवर्तक सूचकांक संवेदनशीलता है, जबकि तापमान संवेदनशीलता दोनों के लिए समान है। हमने एक साथ माप में दो मापदंडों के बीच क्रॉस-सेंसिटिविटी के मुद्दे की भी जांच की है। हमने निष्कर्ष निकाला है कि क्रॉस-सेंसिटिविटी बहुत छोटी है और इसे मापना काफी हद तक अविश्वसनीय है। अंत में, हमने रिंग-कोर फाइबर और ग्रेडेड-इंडेक्स मल्टीमोड फाइबर का उपयोग करके एक स्थिर ऑप्टिकल ट्वीजर जांच का प्रस्ताव दिया है, जो स्थिर OAM मोड का समर्थन करता है।

हमने लीकी गाइडिंग संरचना में मोड और मोडल हानि प्राप्त करने के लिए एक नई विधि प्रस्तावित की है। लीकी मोड को दी गई लीकी संरचना के सीमित-निर्देशित मोड के रूप में प्राप्त किया जाता है। लीकी संरचना के माध्यम से इस मोड को प्रसारित करके हानि प्राप्त की जाती है। मोड गणना और प्रसार हानि को कॉलोकेशन विधि का उपयोग करके किया गया है। हमने कम और उच्च अपवर्तक सूचकांक के नौ आवधिक द्विपरतों के साथ एक ब्रैग फाइबर को बेहतर मोड कारावास के लिए अनुकूलित किया है। संरचना 0.6 माइक्रोन से 1.242 माइक्रोन तक तरंग दैर्ध्य के लिए मोड को अच्छी तरह से सीमित रखती है, बहुत कम हानि के साथ। उच्च-क्रम मोड लीक हो जाते हैं और उन्हें एबीसी सीमा स्थितियों के साथ संभाला गया है।

अंत में, असमान रूप से वितरित मार्गदर्शक क्षेत्रों वाली संरचनाओं के मॉडलिंग के लिए, कॉलोकेशन बिंदुओं को फिर से वितरित करना आवश्यक है। यह एक परिवर्तन फ़ंक्शन के साथ प्राप्त किया गया है जो परिवर्तित और वास्तविक स्थान के बीच एक-से-एक पत्राचार सुनिश्चित करता है। हमने बड़े कोर ब्रैग फाइबर

के लिए मोडल गणना में बेहतर सटीकता का प्रदर्शन किया है। हमने यह भी दर्शाया है कि ठोस कोर ब्रैग फाइबर को भी इस परिवर्तनीय रूपांतरित कोलोकेशन विधि का उपयोग करके कुशलतापूर्वक मॉडल किया जा सकता है।

CONTENTS

SYMBOLS AND THEIR DESCRIPTIONS

Symbol	Description	Symbol	Description
$\boldsymbol{E}$	Electric field	$\phi_{\mathrm{n}}^{l}(r)$	Collocation function
$\boldsymbol{D}$	Electric displacement	$c_n(z)$	Expansion coefficients
$\boldsymbol{H}$	Magnetic field	$\psi(r)$	Radial field variation
$\boldsymbol{B}$	Magnetic intensity	$\Phi(\varphi)$	Azimuthal field variation
$\boldsymbol{E_z}$	z-component of electric field	φ	Azimuthal angle
$\boldsymbol{E_y}$	y-component of electric field	$L_n^l(r)$	Associated-Laguerre functions
$\boldsymbol{E_x}$	x-component of electric field	α	Scaling factor
$\boldsymbol{H_z}$	z-component of magnetic field	χ	Wave propagating without fast-varying phase part
$\boldsymbol{H_y}$	y-component of magnetic field		
$\boldsymbol{H_x}$	x-component of magnetic field	w	Weight factor
ϵ	Permittivity in dielectric medium	Δz	Step-size in propagation
ϵ_0	Permittivity in free space	Γ	Attenuation constant
μ	Permeability of dielectric material	δn	Refractive index modulation
μ_0	Permeability in free space	Λ	Grating period in FBG
c	Speed of light	ξ	Thermo-optic coefficient
ϵ_{r}	Relative permittivity	α_T	Thermal expansion coefficient
μ_{r}	Relative permeability	k_n	Refractive index sensitivity
v	Velocity of light in medium	k_T	Temperature sensitivity
λ	Wavelength	Y	Young's modulus of elasticity
n_r	Reference refractive index	δ_{mn}	Kronecker delta
n_c	Refractive index of core	n_{eff}	Effective refractive index
n_{cl}	Refractive index of cladding	P_{frac}	Fractional power in core
l	Orbital angular momentum number	d	Thickness of ABC layer
∇	Nabla or del operators	g	Gradient constant for GIMF
$\hbar$	Reduced Planck constant	σ	Transformed variable
k_0	Free-space wave vector	$h(\sigma)$	Transformation
ω	Angular frequency	$h'(\sigma)$	First derivative of transformation
β	Propagation constant	$h''(\sigma)$	2nd derivative of transformation
r_c	Radius of core	$U(\sigma)$	Transformed transverse field

ABBREVIATIONS

Abbreviation	Full Form
ABC	Absorbing Boundary Conditions
BPM	Beam Propagation Method
DMD	Differential Mode Delay
FBGs	Fiber Bragg Grating
FD	Finite Difference
FE	Finite Element
FM-FBG	Few-mode Fiber Bragg Grating
FM-SIF	Few-mode Step-Index Fibers
FWHM	Full Width at Half Maximum
GIMF	Graded-Index Multimode Fibers
HOMs	Higher-Order Modes
LEDs	Light-Emitting Diodes
LMA	Large Mode Area
LPGs	Long-Period Grating
MDM	Mode Division Multiplexing
MMFs	Multimode Fibers
MM-SIF	Multimode Step-Index Fibers
OAM	Orbital Angular Momentum
PMP	Polymethyl Pentane
RCFs	Ring Core Fibers
RIU	Refractive Index Unit
SMFs	Single-Mode Fibers
SMS	Single-mode-Multimode-single-mode
SSNP	Split-Step Non-Paraxial
TMM	Transfer Matrix Method
VTCM	Variable Transformed Collocation Method

LIST OF FIGURES

LIST OF TABLES