

**GLOBAL-LOCAL THEORIES FOR
SMART PIEZOLAMINATED PLATES AND SHELLS**

by

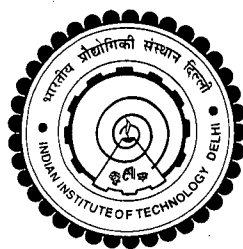
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Submitted

*in fulfillment of the requirements of
the degree of*

Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI
JULY 2012

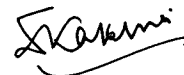
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Khushi



Certificate

This is to certify that the thesis titled “**Global-Local Theories for Smart Piezolaminated Plates and Shells**” being submitted by **Jayanta Kumar Nath** to the Indian Institute of Technology Delhi, New Delhi for the award of the degree of **Doctor of Philosophy** in Applied Mechanics is a record of original bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.



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Acknowledgements

I express my sincere gratitude to Prof. S. Kapuria, my esteemed Guide, for his invaluable guidance and supervision throughout my research work.

My college, Institute of Technical Education & Research, Bhubaneswar, has provided me support and assistance in carrying out my work along with my regular duties. I owe a lot to my college.

I wish to thank my family and friends who have directly or indirectly encouraged me during this period.

Jayanta Kumar Nath .
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Abstract

Piezoelectric materials are finding ever growing applications as distributed sensors and actuators in structural systems for damage detection, vibration suppression, etc. Laminated composite materials, which are now preferred for structural use in aerospace, naval and space structures, are ideally suited to house such sensory and actuator layers, because of their layered construction and compatibility of structural properties. Development of accurate and efficient laminate theories for such hybrid piezoelectric laminated smart structures poses challenge due to high layerwise inhomogeneity in mechanical, electrical and thermal properties.

In this work, a new efficient two-dimensional (2D) laminate theory, namely the improved zigzag theory (IZIGT) is developed for hybrid piezoelectric circular cylindrical shells under electrothermomechanical loading, by extending the zigzag theory for hybrid plates. The electric potential and thermal fields are approximated as piecewise linear across a number of sublayers so that the nonlinear potential field and actual temperature profile across the thickness can be captured to any desired degree of accuracy. The transverse displacement is approximated to explicitly account for the transverse normal strain due to thermoelectric field, without introducing additional unknowns. It accounts for the layerwise variation of inplane displacements, and exactly satisfies the continuity and boundary conditions on the transverse shear stresses at the layer interfaces and the inner and outer surfaces. Yet, the number of primary displacement variables is only five. To investigate the effect of the layerwise variation of inplane displacements, a smeared theory with the same number of displacement unknowns is also developed. The extended Hamilton's principle is used to derive the governing equations of motion and the variationally consistent boundary conditions. The theories are critically assessed for their accuracy for a wide variety of heterogeneous hybrid composite and sandwich laminates in direct comparison with the available exact three-dimensional (3D) piezothermoelasticity solutions for

the static response of simply-supported (i) infinite-length angle-ply cylindrical panels and (ii) finite-length cross-ply shells and shell panels under electrothermomechanical loading, free vibration response and forced vibration response under electromechanical loading. It is shown that the IZIGT is very accurate even for shells with highly inhomogeneous laminates, whereas its smeared counterpart yields highly inaccurate results. For the thermal loading, the explicit inclusion of transverse thermal deformation in the approximation of deflection has worked extremely well in improving the accuracy of the IZIGT not only for the deflection but also for all other response entities.

Accurate prediction of transverse stresses through 2D laminate theories that are efficient, directly from the constitutive equations, has remained a challenge, for both elastic as well as hybrid laminates. The computation from the constitutive equation is desirable since the other alternative of computing by integrating the 3D equations of momentum, requires accurate computation of up to at least third order derivatives of displacements, which is difficult to achieve in the standard finite element formulations.

Two efficient 2D theories, namely the zigzag-local theory (ZLT) and the improved global-local theory (IGLT), are developed for hybrid plates under electrothermomechanical loading, with the purpose of accurately predicting the transverse shear stresses directly from the constitutive equations. This is achieved by superimposing third order local (layerwise) variation on the zigzag approximation (in case of ZLT) and global third order variation (in case of IGLT) of the inplane displacements, resulting in the nine and eleven primary displacement variables, respectively. The theories approximate the electric potential as quadratic across the piezoelectric layers, and retain all other features of the IZIGT. The developed theories are assessed for mechanical, electrical and thermal load cases, by direct comparison with the 3D exact piezothermoelasticity solution, for static and dynamic response of hybrid plates of a wide variety of laminate configurations. The assessment reveals that the ZLT yields accurate predictions of all response entities including the transverse shear stresses for elastic as well as hybrid laminated plates under mechanical and electric potential loading, and is superior to both the IGLT and the zigzag theory. However, under thermal loading, the IGLT performs better than the ZLT and the zigzag theory in predicting the transverse shear stresses.

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