A

Thesis on

Flow Problems in Elastico-Viscous Fluids

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C E R T I F I C A T E

This is to certify that the thesis entitled 'Flow Problems in Elastico-viscous fluids' that is being submitted by Mr. S.P.Gulati for the award of the Degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him under my supervision and guidance. Mr. S.P.Gulati, after taking his D.I.I.T. (Rheology and Plasticity) diploma in 1964 from I.I.T., Kharagpur left for I.I.T., Kanpur as Junior Research Assistant with Prof. J.N.Kapur. After spending one year there he joined I.I.T., Delhi and has worked in the Department of Mathematics since the 7th of August, 1965 and the thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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SYNOPSIS

Due to the paramount importance of artificial materials such as plastics, polymers and synthetic fibres for textile in industries and technology, it is becoming very essential for an engineer to be familiar with special types of problems encountered in routine work. The mechanical response to stress of these materials is quite often different from that of their constituents, the natural products. A description of mechanical response can help us to make an advance assessment of their potential new uses.

The two simple types of mechanical behavior are linear elasticity, governed by Hooke's law; and fluidity, governed by Newton's law of viscosity. The class of fluids which exhibits both elastic and viscous properties is known as elastico-viscous fluids. Solutions of high polymers, pitch and cement etc. are all elastico-viscous fluids.

The investigations of this thesis which is divided into eight chapters comprise of a study of some flow problems for Walters liquid B'' which is an idealised elastico-viscous model with a very short memory and is based on experiments. In chapter I, a systematic survey of elastico-viscous fluids including Walters liquid B'' has been conducted. After this, a brief survey of the problems investigated and the relevant literature has been outlined. The basic equations governing the flow and the heat transfer in a form employed in the following chapters have been recorded towards the end of the chapter.

Chapter II deals with the free convection flow of Walters liquid B'' past a porous flat plate and a porous circular cylinder. Here a new technique has been developed to obtain exact solutions based on the fact that all non-Newtonian flows are perturbations of the corresponding viscous case flows. This technique seems applicable only when the basic viscous solution exists in a closed form and the elastico-viscous equations of motion which are of a higher degree admit a regular perturbation solution, the elastic number being the perturbation parameter. For the free convection elastico-viscous flow past a porous flat plate, it is found that the presence of elasticity decreases the boundary layer thickness and does not affect the skin friction. Also, at any point in the flow region the normal stress difference which is non-zero only in the presence of elastic elements increases with an increase in the elastic number. Again for the free convection elastico-viscous flow past a porous circular cylinder the boundary layer thickness and the skin friction are affected by elasticity exactly in the same way as for the plate problem. The two normal stress differences also increase with an increase in the elastic number. Apart from a complete study of the elastic effects in the problems considered here, the value of the second chapter lies in elucidating a technique of obtaining exact solutions for such non-Newtonian flows which result in the equations of motion having higher degrees than the corresponding viscous case.

Chapter III is concerned with the generalised Couette type flow of Walters liquid B'' between two parallel porous flat plates and a porous annulus. The generalised Couette type flow means the flow due to the motion of one of the boundaries and due to the presence of a constant pressure gradient applied in the direction of flow. Here too, the technique indicated in chapter II has been used to obtain exact solutions. For the generalised Couette type flow between two parallel porous flat plates it is found that a point of separation may exist at the lower plate which is kept fixed and it occurs for a smaller adverse pressure gradient for blowing than for suction in both the viscous and the elastico-viscous cases;

however, for the same amount of blowing or suction the elastico-viscous values are less. Further in the absence of a pressure gradient, the skin friction at the stationary plate is found to increase with an increase in blowing but it decreases with an increase in suction in the viscous as well as the elastico-viscous case; the values in the latter case are always less. The effect of the presence of an adverse pressure gradient is to cause an over all increase in the skin friction. For the parallel problem of an annulus almost same type of results have been obtained. A point of separation may exist at the fixed cylinder for suction as well as injection and for suction the adverse pressure gradient required to provoke separation is greater than that for blowing in both the cases, viscous as well as the elastico-viscous for $-2 < R < 0$ (a range of blowing, R being the suction parameter), the adverse pressure gradient required in the elastico-viscous case is greater, for $R = -2$ it is the same and for all other values of R it is smaller. Again the skin friction at the outer cylinder which is kept fixed is found to increase with an increase in suction but it decreases with an increase in blowing. In the absence of pressure gradient for all positive R, the elastico-viscous values of skin friction are smaller but for negative R upto $R = -2$, they are greater and beyond this value they are once again smaller. The effect of the presence of an adverse pressure gradient is to cause an over all increase in the skin friction.

In chapter IV a study has been made of the flow of Walters liquid B'' near an oscillating flat plate and between two oscillating flat plates which are electrically non-conducting under a uniform transverse magnetic field. An interesting feature of this problem is that the two plates oscillate with any phase difference, same frequency and different amplitudes in the two cases (i) when the magnetic lines of force are fixed relative to the fluid and (ii) when the magnetic lines of force are fixed relative to one of the plates. Expressions for the velocity profile, the induced magnetic field and the shear stresses on the plates in all the situations have been calculated exactly as well as approximately in the two limiting cases of small and large values of the Reynolds number which is based upon the amplitude of oscillations. In case of two oscillating plates the skin frictions on the two plates tend to be equal in the limiting case when the Reynolds number tends to zero. The shearing stress on the lower plate in the limiting case when the Reynolds number tends to infinity behaves as if the other plate were absent. These results hold whether the magnetic lines of force are fixed relative to the fluid or to one of the plates. The effects of elasticity on the skin friction for small values of the Reynolds number at the end of a time period have also been studied through tables and graphs to depict further the difference between the two cases.

In chapter V, the torsional oscillations of an infinite disc in Walters liquid B'' have been considered. The method of investigation is the same as employed by Rosenblat while solving the corresponding viscous case problem and it is that of perturbing the solution in ascending powers of a parameter based on the amplitude of oscillations. The first order solution consists of a transverse velocity and the second order solution gives a radial-axial flow composed of a steady part and a fluctuating part. The steady part of the radial flow does not vanish outside the boundary layer and hence the equations of motion are solved by a Pohlhausen type approximate method for the steady part of the flow. The effect of the elastic elements is to increase the boundary layer to start with and afterwards to decrease it and also to increase the shearing stress at the disc. The steady radial and the steady axial velocity fall short of those in the viscous case in the beginning but afterwards their values lie above.

Chapters VI to VIII consist of unsteady flows under Stokes approximation.

Here the equations of motion are linearised so that the convective part of the inertia force disappears and the convected derivative of the strain rate tensor is replaced by the simple time derivative. The elasticity effects contribute only through the simple time derivative of the strain rate tensor. In chapter VI, the linearised equations of motion for the compressible case have been solved in a general way when the thermal processes are neglected. The corresponding incompressible case solution has been deduced as a limiting case. Making use of this incompressible case solution the motion due to the rotatory oscillations of a spherical shell containing Walters liquid B″ and also the motions due to rotatory and linear oscillations of a solid sphere in an infinite mass of the same fluid have been discussed. For the rotatory oscillations of a spherical shell containing the elastico-viscous fluid and also for the rotatory oscillations of a solid sphere in an infinite mass of the same fluid the elastic elements affect the couple acting on them in the same way. This couple which comprises of two factors, the inertia force and the frictional force is affected by the presence of elasticity in an interesting fashion; the former decreases while the latter increases. In case of linear oscillations of a solid sphere, the drag acting on its surface is also affected in the same way i.e. the factor of inertia force of the drag decreases but that of the frictional force increases due to the presence of elasticity.

In chapter VII, the decay of any initial motion in a spherical vessel both in the compressible as well as the incompressible case and also in a circular cylinder in the incompressible case has been investigated. It is found that the presence of elasticity decreases the modulus of decay of any initial motion in a vessel of a spherical or a cylindrical shape.

Chapter VIII consists of the propagation of sound waves in a compressible elastico-viscous medium comprising of Walters liquid B''. Here the propagation of plane waves when the thermal processes are neglected and also when they are taken into account have been considered. Waves diverging from a spherical surface where a prescribed rotational and also a translational velocity is maintained have also been investigated. It is found that elasticity gives rise to secondary effects. When terms upto first order of the elastic number alone are retained, the elasticity does not affect the amplitude but it quickens the wave propagation, in general.

C O N T E N T S