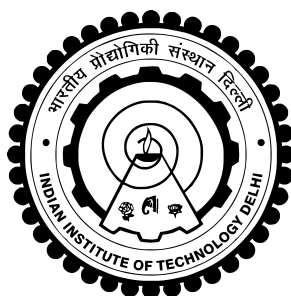


OTFS-BASED CHANNEL ESTIMATION AND MULTIPLE-ACCESS IN DOUBLY SPREAD WIRELESS CHANNEL

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**DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI
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by

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DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

FEBRUARY 2026

Dedicated to
My Family & Friends

Certificate

This is to certify that the thesis entitled “**OTFS-based channel estimation and multiple-access in doubly spread wireless channel**” being submitted by **Mr. Imran Ali Khan** to the Department of Electrical Engineering, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of bonafide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma.

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Acknowledgments

*“Be like a tree and let the dead leaves drop.
Try not to resist the changes that come your way.
Instead, let life live through you.”*

– Rumi

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Imran Ali Khan

Abstract

Orthogonal Time Frequency Space modulation (OTFS) has emerged as a promising candidate for reliable communication in doubly spread wireless environments. Unlike traditional multicarrier systems that operate in the time-frequency domain, OTFS modulates information in the delay-Doppler domain, providing robustness against doubly selective channels. This thesis investigates two critical aspects of OTFS-based communication systems: low-complexity channel estimation and efficient multiple-access techniques.

First, we develop a low-complexity channel estimation algorithm for single-antenna Multi-Carrier OTFS systems employing practical rectangular pulse. The developed method is designed to accurately estimate channel with fractional delays and Doppler shifts by employing two key properties of OTFS modulation: (i) the fine resolution in the delay-Doppler domain that enables separation of individual paths, and (ii) the separability of the effective delay-Doppler domain channel expression into individual delay domain and Doppler domain terms. These features are utilized to formulate an iterative estimation procedure that individually estimates the delay, Doppler shift, and complex gain associated with each multipath component. Unlike existing approaches, the developed method avoids matrix inversion, resulting in significantly reduced computational complexity. Simulation results demonstrate that the algorithm achieves better normalized mean square error performance compared to several existing techniques.

Second, we present a multiple-access technique for Zak-OTFS systems, that supports up-link communication from multiple users having diverse channel characteristics over a shared time-frequency resource, ensuring compatibility with existing communication frameworks. The developed method employs modified transmit and receive filters in the native delay-

Doppler domain of Zak-OTFS to allocate users distinct, non-overlapping time-frequency regions without requiring any guard bands. Analytical evaluation of the resulting time-domain signal confirms that each user's transmission remains confined within its assigned time-frequency region. Numerical results further demonstrate that the developed method effectively suppresses multi-user interference for *sinc* and *root raised cosine filter*, achieving bit error rate and normalized mean square error performance comparable to single-user systems, even under severe Doppler spreads.

The techniques presented in this thesis contribute toward the realization of OTFS systems in next generation networks by addressing the key challenges of channel estimation and multiple access in doubly spread wireless environments.

सार

ऑर्थोगोनल टाइम फ्रीक्वेंसी स्पेस मॉड्यूलेशन (OTFS) द्वि-विस्तारित (doubly spread) वायरलेस परिवेशों में विश्वसनीय संचार के लिए एक आशाजनक तकनीक के रूप में उभरा है। पारंपरिक मल्टीकैरीयर प्रणालियों के विपरीत, जो समय-आवृत्ति (time-frequency) डोमेन में कार्य करती हैं, OTFS सूचना को विलंब-डॉप्लर (delay-Doppler) डोमेन में मॉड्यूलेट करता है, जिससे यह द्वि-चयनात्मक (doubly selective) चैनलों के प्रति अधिक सुदृढ़ बनता है। यह शोध-प्रबंध OTFS-आधारित संचार प्रणालियों के दो महत्वपूर्ण पहलुओं का अध्ययन करता है: निम्न-जटिलता चैनल आकलन तथा कुशल बहु-उपयोगकर्ता अभिगम तकनीकें।

प्रथम, हम व्यावहारिक आयताकार पल्स का उपयोग करने वाली एकल-एंटीना मल्टी-कैरीयर OTFS प्रणालियों के लिए एक निम्न-जटिलता चैनल आकलन एल्गोरिद्म विकसित करते हैं। यह विकसित विधि भिन्नांश विलंब (fractional delay) एवं डॉप्लर शिफ्ट के साथ चैनल का सटीक आकलन करने हेतु OTFS मॉड्यूलेशन के दो प्रमुख गुणों का उपयोग करती है: (i) विलंब-डॉप्लर डोमेन में उच्च विभेदन क्षमता, जो व्यक्तिगत पथों के पृथक्करण को संभव बनाती है, तथा (ii) प्रभावी विलंब-डॉप्लर डोमेन चैनल अभिव्यक्ति की विलंब डोमेन एवं डॉप्लर डोमेन पदों में पृथक्करणीयता। इन गुणों के आधार पर एक पुनरावृत्त (iterative) आकलन प्रक्रिया विकसित की गई है, जो प्रत्येक बहुपथ अवयव से संबंधित विलंब, डॉप्लर शिफ्ट एवं जटिल गेन का पृथक-पृथक अनुमान लगाती है। मौजूदा विधियों के विपरीत, यह विकसित एल्गोरिद्म मैट्रिक्स इन्वर्जन से बचता है, जिससे संगणनात्मक जटिलता में उल्लेखनीय कमी आती है। सिमुलेशन परिणाम दर्शाते हैं कि प्रस्तावित एल्गोरिद्म कई मौजूदा तकनीकों की तुलना में बेहतर सामान्यीकृत माध्य वर्ग त्रुटि (NMSE) प्रदर्शन प्राप्त करता है।

द्वितीय, हम Zak-OTFS प्रणालियों के लिए एक बहु-उपयोगकर्ता अभिगम तकनीक प्रस्तुत करते हैं, जो साझा समय-आवृत्ति संसाधन पर विभिन्न चैनल विशेषताओं वाले अनेक उपयोगकर्ताओं से अपलिंक संचार का समर्थन करती है तथा मौजूदा संचार ढाँचों के साथ संगत बनी रहती है। यह विकसित विधि Zak-OTFS के स्वाभाविक विलंब-डॉप्लर डोमेन में संशोधित प्रेषक एवं ग्राही फिल्टरों का उपयोग कर उपयोगकर्ताओं को भिन्न, गैर-अतिव्यापी समय-आवृत्ति क्षेत्रों में आवंटित करती है, वह भी किसी गार्ड बैंड की आवश्यकता के बिना। परिणामी समय-डोमेन संकेत का विश्लेषणात्मक अध्ययन यह सुनिश्चित करता है कि प्रत्येक उपयोगकर्ता का संचरण अपने निर्दिष्ट समय-आवृत्ति क्षेत्र तक ही सीमित रहता है। संख्यात्मक परिणाम यह भी दर्शाते हैं कि यह विकसित विधि *sinc* तथा *root raised cosine* फिल्टरों के लिए बहु-उपयोगकर्ता हस्तक्षेप को प्रभावी रूप से कम करता है और अत्यधिक डॉप्लर विस्तार की स्थिति में भी एकल-उपयोगकर्ता प्रणालियों के तुल्य बिट त्रुटि दर (BER) एवं सामान्यीकृत माध्य वर्ग त्रुटि (NMSE) प्रदर्शन प्राप्त करती है।

इस शोध-प्रबंध में प्रस्तुत तकनीकें द्वि-विस्तारित वायरलेस परिवेशों में चैनल आकलन एवं बहु-उपयोगकर्ता अभिगम से जुड़ी प्रमुख चुनौतियों का समाधान प्रस्तुत करती हैं तथा अगली पीढ़ी के नेटवर्कों में OTFS प्रणालियों के व्यावहारिक कार्यान्वयन की दिशा में एक महत्वपूर्ण योगदान प्रदान करती हैं।

Contents

Certificate	i
Acknowledgements	ii
Abstract	iv
List of Figures	xi
List of Tables	xv
List of Abbreviations	xvi
1 Introduction	1
1.1 Evolution of Cellular Communication: From 1G to 6G	3
1.2 Waveform for Doubly Spread Channels	7
1.3 Motivation	11
1.3.1 Channel Estimation in MC-OTFS	11

1.3.2	Multiple Access in Zak-OTFS	13
1.4	Research Objectives and Contributions	14
1.5	Thesis Organization	15
2	OTFS-based Systems	17
2.1	Delay-Doppler domain signaling	18
2.1.1	Quasi-periodic structure in DD domain	19
2.1.2	Wireless Channel in the Delay-Doppler Domain	20
2.1.3	Twisted Convolution	21
2.2	Carrier Waveform for OTFS : Pulsone	22
2.3	Multi-Carrier OTFS (MC-OTFS)	26
2.3.1	MC-OTFS Modulation	27
2.3.2	MC-OTFS Demodulation	28
2.4	Zak-OTFS	30
2.4.1	Zak-OTFS Modulation	32
2.4.2	Zak-OTFS Demodulation	33
2.4.3	Predictability of Zak-OTFS I/O Relation	36
2.4.4	Model-Free Channel Estimation	38
2.5	Comparison between MC-OTFS and Zak-OTFS	39
2.6	Conclusion	40

3	A Low Complexity Channel Estimator for MC-OTFS	41
3.1	Introduction	41
3.2	System Model	43
3.3	Channel Estimation Method	45
3.3.1	Motivation	45
3.3.2	Analysis of estimates in (3.15) for single path	49
3.3.3	Channel estimation method for multi-path channels	50
3.3.4	Complexity of the method	53
3.4	Numerical Results	54
3.5	Conclusion	59
4	Multiple Access technique for Zak-OTFS-based systems	60
4.1	Introduction	60
4.2	System Model for Zak-OTFS Multiple Access	62
4.3	Zak-OTFS Multiple Access Method	65
4.3.1	DD domain processing at transmitter	67
4.3.2	DD domain receiver processing at the BS	70
4.3.3	TF localization of a UT's transmit signal	71
4.3.4	Embedded Pilot and Data Frame for Multiple UTs	75
4.4	Numerical Results and Discussions	78

4.5	Conclusion	86
5	Conclusion and Future Work	87
5.1	Conclusion	87
5.2	Future Work	89
5.2.1	Comparison with Cyclic-Prefix OFDM (CP-OFDM)	89
5.2.2	Optimal operating point for Zak-OTFS	93
5.2.3	Extension to Multi-Antenna Systems	94
5.2.4	A Framework for Delay-Doppler Processing	100
5.2.5	Low-Complexity Equalizer	100
5.2.6	Hardware Realization	101
	Bibliography	102
	Appendices	111
A	Proofs for Chapter 2	112
A.1	Derivation of (2.12) and (2.13)	112
B	Proofs for Chapter 3	114
B.1	Asymptotic Orthogonality of Columns of $A(\tau, \nu)$	114
B.2	Proof of Eq. 3.17	115

B.3	Proof of Theorem 3.1	116
C	Proofs for Chapter 4	117
C.1	Proof of (4.3)	117
C.2	Derivation of (4.6)	118
C.3	Proof of Theorem 4.1	118
C.4	Proof of Theorem 4.2	119
	List of Publications	121
	Curriculum Vitae	122

List of Figures

1.1	Evolution of physical layer waveform across generations of cellular communication.	3
2.1	Quasi-periodic structure of a DD domain signal. A localized pulse placed within the fundamental period $\mathcal{D}_0$ is extended across the entire DD plane according to the quasi-periodicity property of the Zak transform, resulting in shifted replicas modulated by complex exponential factors [29].	19
2.2	Pulsone structure in (a) Time-domain (b) Frequency domain	23
2.3	Delay-Doppler domain information lattice over sampled over the fundamental region $\mathcal{D}_0$	25
2.4	MC-OTFS transmitter structure	27
2.5	MC-OTFS receiver structure	29
2.6	Zak-OTFS transmitter structure of s -th UT	32
2.7	Zak-OTFS receiver structure of q -th UT at BS	33
2.8	Quasi-periodic DD domain subframe illustrating the effective channel responses to point pilots transmitted at locations $(k_{p1,q}, l_{p1,q})$ (pink dot) and $(k_{p2,q}, l_{p2,q})$ (blue dot). The ellipses represent the corresponding localized channel responses within the crystalline regime.	36

2.9	Doppler domain aliasing when $\nu_{p,q} < \nu_{ds,q}$	37
2.10	Delay domain aliasing when $\tau_{q,q} < \tau_{ds,q}$	37
3.1	(a), (b) : Channel path separability in DD domain.	47
3.2	Separability of delay and Doppler terms.	48
3.3	A practical OTFS frame as in [39]. One pilot symbol and $(2M_\tau N_\nu - 1)$ guard symbols. Zeroes are transmitted on guard symbols which serve the purpose of reducing interference between data and pilot symbols. Estimation is performed using only the symbols received in the yellow region.	55
3.4	NMSE performance of developed estimation method in comparison with other methods for $M = 64$	55
3.5	NMSE performance of developed estimation method in comparison with other methods for $M = 128$	57
3.6	NMSE performance of developed estimation method in comparison with other methods for $M = 32$	58
3.7	SER performance of developed estimation method in comparison with other methods	59
4.1	Transmitter and Receiver processing in Zak-OTFS-based uplink multi-user system	63
4.2	Non-overlapping TF resource allocation for UTs.	66
4.3	DD domain processing and corresponding TF allocation : (a) Single UT (b) Multiple UTs	69
4.4	Embedded pilot and data frame for q -th UT with data, pilot and guard region .	76

4.5	Leakage ratio $\frac{I_{q,s}}{S_{s,s}}$ (see (4.33)) vs. ν_{max} when (a) only UT-1 is transmitting (b) only UT-3 is transmitting	80
4.6	BER vs. SNR for UT-1 for multi-user allocation and single-user with embedded pilot-data Zak-OTFS frame. Fixed PDR= 0 dB and $\nu_{max} = 815$ Hz for all UTs. LSMR equalization.	81
4.7	NMSE vs. SNR for UT-1 for multi-user allocation and single-user with embedded pilot-data Zak-OTFS frame. Fixed PDR= 0 dB and $\nu_{max} = 815$ Hz for all UTs.	81
4.8	BER vs. PDR for UT-1 for multi-user allocation and single-user with embedded pilot-data Zak-OTFS frame. Fixed SNR $\rho_q = 20$ dB and $\nu_{max} = 815$ Hz for all UTs. LSMR equalization.	83
4.9	NMSE vs. PDR for UT-1 for multi-user allocation and single-user with embedded pilot-data Zak-OTFS frame. Fixed SNR $\rho_q = 20$ dB and $\nu_{max} = 815$ Hz for all UTs.	83
4.10	BER vs. ν_{max} for UT-1 for multi-user allocation and single-user with embedded pilot-data Zak-OTFS frame. Fixed SNR $\rho_q = 20$ dB and PDR = 0 dB for all UTs. LSMR equalization.	84
4.11	NMSE vs. ν_{max} for UT-1 for multi-user allocation and single-user with embedded pilot-data Zak-OTFS frame. Fixed SNR $\rho_q = 20$ dB and PDR = 0 dB for all UTs.	84
4.12	Effective spectral efficiency (SE) vs. ν_{max} for UT-1 for proposed multiuser allocation with different pulse shaping filters. Fixed SNR $\rho_q = 20$ dB and PDR = 0 dB for all UTs. LSMR equalization.	84
4.13	BLER vs. SNR for UT-1 for proposed multiuser allocation with different pulse shaping filters. Fixed PDR = 0 dB, $\nu_{max} = 815$ Hz and $\tau_{max} = 2.51\mu s$ for all UTs. LSMR equalization.	84

5.1	Zak-OTFS (Embracing ICI) vs. OFDM (avoiding ICI).	90
5.2	Three different Zak-OTFS frame structures for pilot-data allocation. Each frame is M taps wide along the horizontal delay axis and is N taps wide along the vertical Doppler axis. In each frame structure, the red dot in the middle depicts the pilot pulsons used for acquiring the effective discrete DD domain channel filter. The received pulsons in the yellow colored “pilot region” are used for the estimation. The width of this pilot region should be at least $k_{max} = \lceil B\tau_{max} \rceil$ taps in order that a significant fraction of the received pilot energy is used for channel estimation. The dark blue colored “data region” carry information and those in the white colored “guard region” are used to avoid interference between the data and the pilots. The three different frame structures differ in the width of the pilot region.	91
5.3	Ratio of effective SE achieved by Zak-OTFS to that achieved by CP-OFDM. .	93
5.4	MIMO-OTFS Block Diagram	95
5.5	Illustration of an OTFS frame for a MIMO setup. Each transmit antenna employs one pilot symbol, resulting in N_t pilot symbols in total, along with $(M_\tau N_\nu)$ guard symbols per pilot. Zero symbols are transmitted over the guard regions to suppress interference between data and pilot symbols. Channel estimation for each transmit-receive antenna pair is performed using only the received symbols within the corresponding yellow pilot region.	98

List of Tables

1.1	Key Physical Layer Innovations Across Generations [8]	2
3.1	Complexity of OTFS Channel Estimation Methods	54
4.1	Simulation parameters for multiple UTs	79
4.2	Power-delay profile of Veh-A channel model	79
5.1	Comparison between OFDM and ZAK-OTFS for Next Generation Wireless Communication	91

Glossary

1G	First Generation
2G	Second Generation
3G	Third Generation
3GPP	3rd Generation Partnership Project
4G	Fourth Generation
5G	Fifth Generation
6G	Sixth Generation
AI	Artificial Intelligence
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
BS	Base Station
CDMA	Code Division Multiple Access
CP	Cyclic Prefix
CP-OFDM	Cyclic Prefix Orthogonal Frequency Division Multiplexing
CSI	Channel State Information
DD	Delay-Doppler
DSS	Dynamic Spectrum Sharing
DZT	Discrete Zak Transform
EDGE	Enhanced Data Rates for GSM Evolution
eMBB	Enhanced Mobile Broadband
FDM	Frequency Division Multiplexing
FDMA	Frequency Division Multiple Access
FeMBB	Further Enhanced Mobile Broadband

FM	Frequency Modulation
Gbps	Gigabits per second
GMSK	Gaussian Minimum Shift Keying
GPRS	General Packet Radio Service
GSM	Global System for Mobile Communications
HSDPA	High Speed Downlink Packet Access
HSUPA	High Speed Uplink Packet Access
i.i.d	Independent and Identically Distributed
I/O	Input/Output
ICI	Inter Carrier Interference
IMT	International Mobile Telecommunications
ISAC	Integrated Sensing and Communication
ISI	Inter Symbol Interference
JSAC	Joint Sensing and Communication
LTE	Long Term Evolution
MA	Multiple Access
Mbps	Megabits per second
MC-OTFS	Multi-carrier Orthogonal Time Frequency Space
MIMO	Multiple Input Multiple Output
MIMO-OFDM	MIMO Orthogonal Frequency Division Multiplexing
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimate
mMTC	Massive Machine Type Communication
mmWave	Millimeter Wave
MUI	Multi User Interference
muRLLC	Massive Ultra Reliable Low Latency Communication
NMSE	Normalized Mean Square Error
NOMA	Non-Orthogonal Multiple Access
OMA	Orthogonal Multiple Access
OMP	Orthogonal Matching Pursuit
SBL	Sparse Bayesian Learning
SER	Symbol Error Rate
TF	Time-Frequency