SOME SINGULAR PERTURBATION PROBLEMS IN HYDROMAGNETICS AND HYDRODYNAMICS

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Indian Institute of Technology, Delhi

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A THESIS ON
SOME SINGULAR PERTURBATION PROBLEMS
IN HYDROMAGNETICS AND HYDRODYNAMICS

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Submitted to the Indian Institute of
Technology, New Delhi for the award
of the Degree of Doctor of Philosophy
in Mathematics
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C E R T I F I C A T E

This is to certify that the thesis entitled "Some Singular Perturbation Problems in Hydro-magnetics and Hydrodynamics" which is being submitted by Mr. Shanti Swarup for the award of degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

MPS

(M.P. Singh),

Nov. 27 , 1972.

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CONTENTS

SYNOPSIS

During the recent years the study of interaction between magnetic field and moving electrically conducting fluids acquired considerable attention in view of its importance in the fields of science and technology. The study of the motion of an electrically conducting fluid in the presence of a magnetic field has given rise to a subject, viz., Magneto-hydrodynamics. Magneto-hydrodynamics finds important applications in Astronomy and Geophysics, stellar and solar fields, dynamo theory, conversion of energy, MHD generators, ion propulsion etc,

The flow of fluids through porous media has become sufficiently wide to find application in many branches of engineering and applied science, in such diverse fields as ground water and hydrology, petroleum engineering, soil mechanics etc. The general theory of the flow of fluids through porous materials can be considered as the branch of scientific treatment of the problems of irrigation, soil erosion, and tile drainage. Most naturally and artificially porous materials have a random void structure. The structure of such materials can be described, in fact, in statistical terms. Even so, it is possible to treat the flow of fluids through such materials on a macroscopic basis.

The purpose of this thesis is to study some theoretical aspects of some flow problems in hydromagnetics and fluid mechanics of permeable medium, which can be solved mathematically, by the method of matched asymptotic expansions. The thesis comprises Six Chapters.

In the first Chapter of the thesis, a critical survey of the literature has been carried out. It consists of three sections; the first two sections includethe basic concepts, governing equations, definitions and the mathematical and physical aspects of the flow of an electrically conducting fluid in the presence of a magnetic field, through porous media and the theory of heat and mass transfer. The third section of this Chapter contain the method of solutions, i.e., the basic concepts of the theory of singular perturbation and its subsequent development pertaining to the problems.

The second Chapter deals with the slow, steady flow of an incompressible, viscous, electrically conducting fluid past a sphere in the presence of a uniform magnetic field. The solution is, sought, by using the method of matched asymptotic expansions. The applied magnetic field is weak and is aligned along the free stream direction. The basic assumptions are that the magnetic pressure number $s(= \frac{M^2}{R_m R_e})$

is of $O(1)$ and $R_m << R_e$. The solution requires the matching of two asymptotic expansions at the surface of the body but not at infinity while the other satisfy the condition at infinity but not at the surface of the body. The influence of R_m and R_e on the drag force is studied upto order $O(R_m R_e)$. The contribution from the Maxwell stress is absent because it produces symmetric changes in the drag force.

The third Chapter deals with the slow, steady flow of an incompressible, viscous, electrically conducting fluid past a circular cylinder in the presence of a uniform magnetic field. The applied magnetic field is weak and is aligned along the free stream direction. The basic assumptions made here are that the magnetic pressure number $M(=\mu_e H_o a(\frac{\sigma}{\mu})^{\frac{1}{2}})$ is of $O(1)$ and $R_e << M$, $R_m << M$. The solution is sought by the method of matched asymptotic expansions. The influence of magnetic field and viscosity on the drag force is studied upto the order $O(\frac{Re}{(\log M)^2})$ and $O(\frac{R_m}{\log M})$. It is found that there is no contribution of the higher order terms on the drag except for the first term, the reason being that changes produced in the stresses due to these terms are symmetric.

The fourth Chapter deals with the mixed convection problem past the porous sphere at small Reynolds and Grashof numbers. The problem is formulated by using the usual Navier-Stokes equations describing the flow outside the sphere while inside the sphere the flow is governed by Darcy law. The undisturbed fluid is moving at speed U_∞ with regard to the sphere and the direction of the uniform stream is taken along the positive X-axis, i.e. opposite to the direction of gravity. The basic assumptions of the present analysis is restricted to a medium of unit Prandtl number together with $Re < O(1)$ and $G < O(1)$ such that $G = o(Re^2)$ as $Re \to o$ where $\epsilon = \frac{G}{Re^2}$. The solution is determined by the method of matched asymptotic expansions involving three simultaneous expansions: interior, inner and outer. The effect of the permeability on the drag is studied upto an $O(Re^2 \log Re)$ in forced convection and upto an $O(\epsilon Re^2 \log Re)$ in free convection. Permeability affects both the Nusselt number and the drag force through the forced convection as well as the natural convection. The drag force decreases as the permeability, K increases. In the case of free convection, the permeability, K tends to oppose the forced convection effect while if $t_w < t_\infty$ (G negative), the role of the permeability, K is now reversed. The drag force decreases with the permeability, K in the range $0 \leq K \leq 3$ with $G = 0, \pm 10^{-6}, \pm 10^{-4}, \pm 10^{-2}$ and .001

$\leqslant Re \leqslant 0.7$. The influence of the permeability K is to decrease the Nusselt number: Nu, both through the forced and free convection effects and the extent of decrease becomes smaller as K increases. If $t_w < t_\infty$, the role of K through the free convection is that it tends to oppose the forced convection effects.

The fifth Chapter is concerned with the problem of slow, steady flow of an incompressible, viscous, electrically conducting fluid, carrying a uniform current past a porous non-conducting sphere of arbitrary conductivity. The applied magnetic field is weak and an aligned current of uniform density J_o and conductivity σ_o^* is passed. The basic assumptions are that the magnetic pressure number $R_h(=\frac{\mu_e J_o^2 a^2}{\rho U_\infty^2})$ is of O(1) and Re << 1. The effect of permeability, K on the drag upto O(Re) is equivalent to the reduction of the free stream velocity and the current density by the factors (2+K) and $\sqrt{2+K}$ respectively. It is seen that the permeability as well as the current both tend to increase the length of eddies.

The Sixth Chapter consists of two main sections: the first section concerns with the problem of the effect of permeability on the mass transfer from a porous sphere of radius a immersed in a uniform stream of incompressible viscous fluid with concentration of the reacting species

C_∞ and velocity $U_\infty\hat{i}$ in Stokes flow with second order surface reactions. The construction of the asymptotic expansions is the same as that for the flow field. The basic assumptions of the present analysis are $Re << 1$ and the Prandtl number, Pr, to be of order unity. The average Sherwood number, Sh is calculated in power of Peclet numbers, Pe upto the order $O(\epsilon^2)$, where $\epsilon = \frac{1}{2}$ Pe. We notice that $2\alpha\left\{= [-(2+\sigma) + [(2+\sigma)^2 - 4\ ]^{\frac{1}{2}}]\right\}$ is a negative dimensionless parameter which depends on mass diffusivity, reaction rate constant and free stream concentration of the reacting species. The convective mass diffusion will tend to increase the Sherwood number. The permeability effect appears in the terms of $O(\epsilon^2 \log\epsilon)$ which tends to reduce the Sherwood number. The last section of the Chapter concludes with the discussion of the effect of permeability on the mass transfer from a porous circular cylinder which is placed in an otherwise parallel uniform stream of viscous, incompressible fluid with concentration of the reacting species C_∞ and velocity $U_\infty\hat{i}$ with second order surface reaction. The flow inside the cylinder is governed by the Darcy law whereas outside the cylinder, it is governed by the usual Navier-Stokes equations. The results show that the permeability does not affect the Sherwood number, Sh upto an $O(\frac{1}{(\log Re)^3})$ because the convective effects are transcendentally small near the body in the two dimensional case.