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ON STOCHASTIC OPTIMAL ESTIMATION FOR DISTRIBUTED PARAMETER SYSTEMS

by

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ABSTRACT

This thesis is concerned with the filtering and smoothing of noisy measurement data obtained from linear and nonlinear distributed parameter systems, that are subject to stochastic disturbances. Linear filtering problem for distributed parameter systems has received considerable attention in recent years. In the present study, solution of this problem is obtained by applying two new techniques. The first of these consists in using minimum principle to obtain optimal gain matrix for minimum variance filter under the requirement that the estimates are unbiased. This direct approach to Kalman-Bucy type distributed filters indicates that these filters are optimal for a variety of performance criteria. In the second technique an extension of innovations approach to distributed parameter systems is carried out to provide an appealingly simple derivation of Kalman-Bucy type distributed filters. A special feature of the present study lies in the consideration of manifold possibilities arising in distributed state estimation problems. Thus Kalman-Bucy type filters are established for processing measurement data obtained either from the interior of the system, that are spatially-continuous or spatially-independent or

spatially-discrete or else obtained from system boundary Another aspect of analysis presented in this thesis is the study of systems corrupted with interior disturbances or simultaneously corrupted with interior and boundary disturbances. In order to tackle estimation problems for systems in which disturbances are entering through boundary conditions a new procedure based upon the extended definition of the operator has been adopted. This helps in generalising all the known estimation theory results for classes of systems with interior disturbances only to classes of systems having both interior and boundary disturbances.

In comparison with filtering, the smoothing solutions for distributed parameter systems are more difficult to obtain. Existing solutions for distributed parameter smoothing problem obtained by extending Kwakernaak's procedure or by developing a two-filter solution similar to that of Mayne do not represent a convenient form of smoothing algorithms. However, since a unified treatment of the smoothing problems for lumped processes, via innovations technique, is known to be simpler in many respects, a study of distributed parameter smoothing problems, via innovations technique is presented. In this way a general formula for smoothing in distributed processes is obtained. New algorithms are derived from the general formula by considering distributed versions of three classes of smoothing problems. The derived algorithms resemble (in form) the well known results of lumped parameter smoothing theory.

Due to the nonavailibility of results in the area of stochastic calculus for distributed case, a direct extension of innovations technique to non-linear distributed parameter systems does not seem to be possible. As such, consideration is given to nonlinear distributed parameter filtering problem from the viewpoint of weighted least-squares estimation over the spatial domain of the system and the time interval of measurement process. Concept of dynamic programming is employed to get a partial differential equation for least-squares estimation. This is solved by a novel linearization technique to generate a second order filtering algorithm. This result which turns out to be a distributed version of Detchmendy-Sridhar filter is believed to be better than a previously suggested solution in terms of linearized Kalman-Bucy type distributed filter.

A solution of nonlinear smoothing problem in the context of least-squares estimation is taken up next. A previous work in this connection suggested the use of differential dynamic programming to solve a two-filter form of the smoothing solution obtained by employing the concept of likelihood functional. In contrast to this, it is shown here that converting the least-squares estimation problem to a two-point boundary-value problem provides a basis for a systematic development of nonlinear smoothing theory. This has led to the establishment of approximate algorithms for fixed-interval, fixed-point and fixed-lag class of smoothing problems.

Examples are presented in which filtering and smoothing of noisy measurement data obtained from specific distributed parameter systems is illustrated. The effectiveness of the theory is brought out by providing numerical results that have been obtained by simulating system and estimator equations on an ICL 1909 digital computer.

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