

**AN ANALYSIS OF TWO AND THREE  
DIMENSIONAL STRESS STATE IN SINGLE  
EDGE CRACKED PLATE**

by

**PRABHAT KUMAR SINGH M.TECH**

A thesis Submitted to the  
Indian Institute Technology, Delhi  
for the award of the degree of

**DOCTOR OF PHILOSOPHY**



**Department of Mechanical Engineering  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI**

**1986**

FOR

ONE

WHO

INSPIRED

## ABSTRACT

A two dimensional finite element model is proposed which allows an accurate numerical evaluation of the crack tip displacement and stress-state for a plane strain tensile loading condition. The model is further extended to include <sup>an</sup> elastic-plastic state of material around the crack tip. Numerical results have been presented in this thesis for the displacements and stresses around the crack tip for the applied nominal stresses of various magnitudes. Solutions were also obtained for the angular and radial variations of stresses around the crack tip. These results have been compared with ~~the~~ Westergaard solution. The FEM result compared well with the theoretical result. The values of the stress intensity factor,  $K_I$ , (for Mode I situation) have been obtained by this model with reasonable accuracy. The elastic-plastic model offers to present the spread of the plastic zone in the vicinity of the crack tip for stepwise incremental loading.

Experiments have been conducted to find out the Mouth Opening Displacement (MOD) values and it compared well with the FEM results.

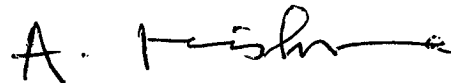
A three dimensional FEM model has also been developed and numerical results have been presented for all the six components of stresses around the crack front. The magnitude of stresses show wide variations around the crack front.

The result of the present investigation should aid in the interpretation of the test data for the edge cracked specimens.

## CERTIFICATE

This is to certify that the thesis entitled, "An Analysis of Two and Three Dimensional Stress State in Single Edge Cracked Plate", being presented by Mr. Prabhat Kumar Singh, is worthy of consideration for the award of the degree of 'Doctor of Philosophy' and is a record of the original bonafide research work carried out by him under my guidance and supervision and that the results contained in it have not been submitted in part or full to any University or Institute for the award of any degree or diploma.

I certify that he has pursued the prescribed course of research.



(Prof. A. MISHRA)  
Department of Mechanical Engg.,  
Indian Institute of Technology,  
New Delhi-110016, INDIA.

May, 1986

## ACKNOWLEDGEMENTS

It is with the greatest of pleasure, that I acknowledge the contribution to this work of my Supervisor, Dr. A. Mishra, Professor, Mechanical Engineering Department, Indian Institute of Technology, New Delhi. I thank him for his expert advise and constant encouragement.

The author is grateful to Dr. F. Sinha for his valuable discussion and assistance in the numerical aspects of this work especially for the use of computer program. I thank him for inspiring encouragement and good will.

He thankfully acknowledges Mr. P.N. Sinha for his cooperation during research work.

Acknowledgement is due to Computer Centre and Department of Applied Mechanics for the facilities provided for the work.

He wishes to acknowledge the help and encouragement received from his many friends and well wishers who went a long way towards providing a favourable atmosphere for this work.

The help and for bearance of my great family are duly acknowledged.

Special acknowledgement is due to wife Rita who sacrificed much during the course of the research effort. Without her understanding and patience this work would not have been accomplished.

He gratefully acknowledges the help and encouragement from Dr. R.K. Jain, Lt. Col. B.D. Singh, Maj. S.R. Singh, Mr. Ramesh Chandra and Amar.

Thanks are also due to Mr. Sarswat for Tracing work and Mr. V.P. Gulati for typing the thesis.

May, 1986:

  
(Prabhat Kumar Singh)

## CONTENTS

		PAGE
CHAPTER-I	INTRODUCTION	1
CHAPTER-II	LITERATURE REVIEW	7
	2.1 INTRODUCTION	7
CHAPTER-III	FINITE ELEMENT ANALYSIS OF TWO DIMENSIONAL STRESS STATE IN SINGLE EDGE CRACKED PLATE	19
	3.1 INTRODUCTION	19
	3.2 STAGE IN THE DEVELOPMENT OF FINITE ELEMENT PROGRAM FOR TWO DIMENSIONAL CRACK ANALYSIS	21
	3.3 ELEMENT DEVELOPMENT	22
	3.4 SINGULARITY ELEMENT	26
	3.4.1 General	26
	3.4.2 Singularity Condition for Cubic Isoparametric Element	26
	3.5 FINITE ELEMENT APPROACH (FEM)	29
	3.5.1 Solution Procedure	31
	3.6 PARTIAL STIFFNESS METHOD	31
	3.7 TEST PROBLEM FOR CUBIC ISOPARA- METRIC SINGULAR ELEMENT	32
	3.8 ANALYSIS OF ELASTIC-PLASTIC INTERFACE	33
	3.9 TEST OF CONVERGENCY FOR FINITE ELEMENT MODEL	35
	3.10 STRESS INTENSITY FACTOR	36
	3.11 ELASTIC-PLASTIC AND MOUTH OPENING DISPLACEMENT (MOD) ANALYSIS OF SINGLE EDGE CRACK	38

## CONTENTS (CONTD.)

	PAGE
CHAPTER-IV	
FINITE ELEMENT ANALYSIS OF THREE DIMENSIONAL STRESS STATE IN SINGLE EDGE CRACKED PLATE	49
4.1 INTRODUCTION	49
4.2 THREE DIMENSIONAL FINITE ELEMENT APPROACH	50
4.3 3D ISOPARAMETRIC BRICK ELEMENT (20 NODES)	53
4.4 3D SINGULARITY ELEMENT	58
4.5 DEVELOPMENT OF THREE DIMENSIONAL FINITE ELEMENT PROGRAM	59
4.5.1 3D Finite Element Model Without Crack	59
4.5.2 3D Finite Element Model With Crack	62
4.5.3 Elastic 3D Stress State of a Single Edge Cracked Plate	63
CHAPTER-V	
EXPERIMENTAL METHOD MEASUREMENT OF MOUTH OPENING DISPLACEMENT (MOD)	67
5.1 GENERAL	67
5.2 SPECIMEN PREPARATION	68
5.3 MOUTH OPENING DISPLACEMENT	68
5.4 CALIBRATION OF CLIP GAUGE	69
5.5 MEASUREMENT OF SPECIMEN SIZE	73
5.6 SET UP FOR MOUTH OPENING DIS- PLACEMENT (MOD) MEASUREMENT	73
5.7 MOD MEASUREMENT UNDER TENSILE LOADING	73
CHAPTER-VI	
RESULTS AND DISCUSSION	76
6.1 INTRODUCTION	76

## CONTENTS (CONTD.)

	PAGE	
6.2	TEST OF CONVERGENCY FOR CRACK PROBLEM FOR APPLIED NOMINAL STRESS OF 5 MN/m <sup>2</sup>	78
6.2.1	X-Displacement	78
6.2.2	Y-Displacement	78
6.2.3	Elastic Variation of Stresses Around the Crack Tip	80
6.2.4	Results for Applied Nominal Stress of 10 MN/m <sup>2</sup>	84
6.2.5	Conclusion	88
6.3	ELASTIC-PLASTIC DISPLACEMENT AND STRESS-STATE	92
6.4	STRESSES ALONG RADIAL LINE EMANATING FROM CRACK TIP	99
6.5	STRESS INTENSITY FACTOR	113
6.6	FINITE ELEMENT ANALYSIS OF THE PLASTIC ZONE	115
6.7	COMPARISION OF MOUTH OPENING DISPLACEMENT (MOD) VALUES OBTAINED BY EXPERIMENTAL AND FINITE ELEMENT METHOD	119
6.8	3D ELASTIC STRESS-STATE FOR EDGE CRACKED PLATE	125
CHAPTER-VII	CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK	137
7.1	CONCLUSIONS	137
7.2	SUGGESTIONS	138
	REFERENCES	139

## CONTENTS (CONTD.)

	PAGE	
APPENDIX-A1	DERIVATION FOR SINGULARITY ELEMENTS	
A1.1	INTRODUCTION	146
A1.2	DERIVATION FOR NODES PLACING	146
A1.3	DERIVATION OF CARTESIAN CO- ORDINATE, JACOBIAN AND $\text{DET} J $ FOR SINGULAR ELEMENT	150
A1.4	CLOSURE	154
APPENDIX-A2:	DERIVATION OF ELASTIC-PLASTIC STRESS- STRAIN MATRIX FOR VON MISES MATERIAL	155
A2.1	INTRODUCTION	155
A2.2	ELASTIC STRESS-STRAIN MATRIX	155
	A2.2.1 Elastic-Plastic Strain Matrix	156
	A2.2.2 Plane Strain Elastic- Plastic Matrix	162
APPENDIX-A3:	FLOW DIAGRAMS FOR 2D AND 3D CRACK ANALYSIS	163
A3.1	INTRODUCTION	163
A3.2	MAIN FEATURE OF THE PROGRAM	163
A3.3	COMPUTER PROGRAM ORGANISATION	164