SOME ASPECTS OF PIECEWISE LINEAR RESISTIVE NETWORKS

by

V. PREM PRAKASH

ELECTRICAL ENGINEERING DEPARTMENT

Submitted
in fulfilment of the requirements
of the degree of

DOCTOR OF PHILOSOPHY

to the

INDIAN INSTITUTE OF TECHNOLOGY, DELHI

1987

R.S.

IN PROFOUND MEMORY OF

MY BELOVED FATHER

P.B.V.V.RAMANA MURTY GARU

CERTIFICATE

Certified that the research work 'Some Aspects of Piecewise Linear Resistive Networks' by V.Prem Prakash, has been carried out under my supervision at the Indian Institute of Technology, New Delhi, and that this work has not been submitted elsewhere for the award of a Degree.

V.C. Prasad

(V.C. PRASAD)
Department of Electrical Engineering
Indian Institute of Technology
New Delhi 110016

ACKNOWLEDGEMENT

The author wishes to express his deep feelings of gratefulness to Dr. V.C. Prasad, Department of Electrical Engineering, Indian Institute of Technology, New Delhi, for his invaluable guidance, perspective supervision and deep inspiration during the course of this work. His suggestions in visualizing the project and sustained interest to attain the objectives envisaged in the project are gratefully acknowledged.

It is a pleasure to acknowledge the help rendered and the keen interest shown by Prof. P.S. Satsangi, Department of Electrical Engineering, Indian Institute of Technology, New Delhi, during the course of this work. The author has benefited very much from a number of courses offered by Prof. Satsangi during the research programme.

The author gratefully acknowledges the help rendered by the Director and the authorities of Dayalbagh Educational Institute, Dayalbagh, Agra, for granting study leave to pursue his Doctoral Programme.

Thanks are due to Mr. J. Anand Rao for his patience and excellent typing of the manuscript and Mr. S.C. Srivastava, for preparing neat drawings. The help rendered by Mr. H.L. Narang and Mr. D.G. Rao in processing this work is sincerely appreciated.

The author wishes to acknowledge his thanks to his wife Shashi, daughter Pinky and son Vimal for their patience and co-operation during his absence from home.

Humble respects are due to his mother Mrs V.Suryakantham who has been a constant source of inspiration, particularly during the pessimistic facets of the work, and with whose blessings this work could be completed.

Lastly, the author wishes to express his sincere thanks to Mrs Shyamala Prasad and Dr. Prasad for extending their kind hospitality during his stay at Delhi.

ABSTRACT

This thesis deals with Piecewise Linear (PL) functions which arise in the study of nonlinear resistive networks. Such networks are characterised by an equation of the form :

$$\underline{f}(\underline{x}) = \underline{y} \qquad (1)$$

where $\underline{f}$ is a continuous piecewise linear function from R^n into itself, $\underline{x}$ is an n-vector of network variables and $\underline{y}$ is an n-vector of inputs. The $\underline{x}$-space is divided into a finite number of convex regions, r , of dimension n , separated by a finite number of (n-1) dimensional hyperplanes. In any region R_m , equation (1) is of the form :

$$\underset{\sim}{J}^{(m)} \underline{x} + \underline{w}^{(m)} = \underline{y} \qquad (2)$$

where $\underset{\sim}{J}^{(m)}$ is the constant Jacobian matrix in R_m and $\underline{w}^{(m)}$ is a constant vector.

This work is concerned with the existence, uniqueness and determination of solutions of piecewise linear resistive networks. The subject matter of this thesis is divided into ten chapters.

The <u>first chapter</u> surveys the existing literature on these problems and the required definitions and notations are presented.

In the <u>second chapter</u>, a method is proposed to solve the so called 'corner problem' which arises in the solution curve

method used to determine one or more solutions of the piecewise linear equations. Let L be the line whose solution curve hits the corner point $\underline{x}^{(p)}$. Let $\underline{y}^{(p)+}$ and $\underline{y}^{(p)-}$ be two points on L in an arbitrarily small neighbourhood of $\underline{y}^{(p)}$, the image of $\underline{x}^{(p)}$. Using the solution curve method determine one or more solutions of $\underline{y}^{(p)+}$ and $\underline{y}^{(p)-}$. The solution curve of L enters the regions in which these solutions lie. Unlike the methods available in the literature, this method, in principle, gives all the extensions of the solution curve in a systematic way. Further this is easily adopted to a.c. networks.

In the next four chapters degree theory and homotopy invariance are used to answer the questions of existence and uniqueness of solutions. Apart from the general formulation $\underline{f}(\underline{x}) = \underline{y}$, the following formulations are also studied

$$\underline{f}(\underline{x}) = \underline{g}(\underline{x}) + \underset{\sim}{H}\,\underline{x} \tag{3}$$

$$\underline{f}(\underline{x}) = \underset{\sim}{A}\,\underline{g}(\underline{x}) + \underset{\sim}{B}\,\underline{x} \tag{4}$$

$$\underline{f}(\underline{x}) = \underline{F}(\underline{x}) + \underline{g}(\underline{x}) \tag{5}$$

where,

- $\underline{g}$ is a continuous piecewise linear function (not necessarily diagonal) from R^n into itself.

- $\underline{F}$ is a continuous piecewise linear function which is diagonal.

$\underset{\sim}{H}$, $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are nxn real matrices. Not many results are available on eqn. (5) in the existing literature. In this thesis we have provided a number of results on this eqn. to solve the existence, uniqueness and determination problems.

The other special features of the results of this thesis are as follows :

(i) $\underline{g}$ is not necessarily diagonal

(ii) controlled sources are allowed even when $\underline{g}$ is nondiagonal.

(iii) zero slopes in the piecewise linear characteristics (equality signs in the inequality conditions for nondiagonal $\underline{g}$) and singular network matrices are simultaneously allowed.

In chapter III the existence of solutions of networks without controlled sources in the linear part are studied. For such networks $\underset{\sim}{H}$ is positive semidefinite and ($\underset{\sim}{A}$, $\underset{\sim}{B}$) is a passive pair. Since most devices in practice satisfy an eventual passivity type of condition, eqn. (3) has atleast one solution if the piecewise linear function is norm coercive. This condition can be waived if a suitable submatrix of $\underset{\sim}{H}$ is positive definite. In the case when $\underline{g}$ is diagonal, an alternative condition is to have a nonsingular $\underset{\sim}{H}$. Conditions for the existence of atleast one solution for eqn. (4) also requires an eventual component-wise strict passivity type of condition on $\underline{g}$. These results are true even if $\underline{g}$ is diagonal or block diagonal.

In chapter IV, we remove the constraint on controlled sources imposed in chapter III. Thus eqn. (2) is shown to have atleast one solution if $\underset{\sim}{H}$ is P_0 , each component of $\underline{g}$ (not necessarily diagonal) satisfies an eventual passivity type of condition and $\underline{f}$ is norm coercive. Various modifications and alternatives of this are discussed in this chapter. In the most general case when $\underset{\sim}{H}$ is not P_0 , eqn. (3) can still be shown to have atleast one solution if $\underset{\sim}{H}^T \underline{g}(\underline{x})$ satisfies either a certain eventual passivity or an eventual component-wise passivity type of condition and $\underset{\sim}{H}$ is nonsingular. Eqn. (4) has atleast one solution if ($\underset{\sim}{A}$, $\underset{\sim}{B}$) is of class W_0 and each component of $\underline{g}$ (not necessarily diagonal) satisfies a certain eventual strict passivity type condition. More generally $\underline{f}$ is surjective if $\underset{\sim}{B}^T \underset{\sim}{A} \underline{g}(\underline{x})$ satisfies an eventual passivity condition and $\underset{\sim}{B}$ is nonsingular. Existence results for equations of the form (5) are not available in the existing literature. This is done in this work using eventual passivity and eventual component-wise passivity type of properties of $\underline{F}$ and $\underline{g}$. Various types of passivity conditions are important as different characteristics satisfy different passivity conditions. For the most general form, namely, eqn. (1), atleast one solution exists if $\underset{\sim}{J}^{e(i)}$, the effective part of the Jacobian matrix of an unbounded region R_i belongs to class P for all i = 1, 2, ... , p.

In chapter V the homeomorphism problem of the networks mentioned in chapter III is studied. It is shown that all the results of chapter III guarantee uniqueness also provided that the Jacobian determinant is nonzero and has the same sign in all the regions (we will henceforth refer to this as the 'determinant sign condition'). Thus the determinant sign condition is almost necessary as well as sufficient from a practical point of view. It is shown that the 'determinant sign condition' can be substituted by monotonicity properties of the piecewise linear resistors. It is important to note that in these derivations strict monotonicity is not necessary. Similar results are true for diagonal and block diagonal $\underline{g}$ also. For eqn. (4) also homeomorphism results are derived using eventual strict componentwise passivity and monotonicity (not necessarily strict monotonicity) type of properties of $\underline{g}$. In the diagonal case the following theorem is proved. $\underline{f}(\underline{x})$ in equations (3) and (4) is a homeomorphism if and only if $\underline{g}(\underline{x}) \neq \underline{g}(\underline{x} + \underline{\alpha})$ for all $\underline{x}$ and every $\underline{\alpha}$ where $\underline{\alpha}$ is a null vector of $\underset{\sim}{H}$ and $\underset{\sim}{B}$ in eqns. (3) and (4) respectively. This can be viewed as a matrix interpretation of the well known result of Desoer and Wu [36] for networks without controlled sources. Unlike their result, this is applicable to some networks containing controlled sources (Example 5.18).

The next chapter i.e., chapter VI deals with the same networks considered in chapter IV, but uniqueness problem is studied. Most of the results of chapter IV are shown to guarantee homeomorphism also if the 'determinant sign condition' is also satisfied. Several new homeomorphism results are derived using monotonicity type properties of $\underline{g}$ and assuming $\underset{\sim}{H}$ to be P_0 or $(\underset{\sim}{A}, \underset{\sim}{B})$ to be a W_0 pair. Further $\underline{f}$ is a homeomorphism if $\underset{\sim}{H}^T \underline{g}(\underline{x})$ or $\underset{\sim}{B}^T \underset{\sim}{A}\, \underline{g}(\underline{x})$ satisfy monotonicity or componentwise monotonicity type of properties and $\underset{\sim}{H}$ or $\underset{\sim}{B}$ is nonsingular in equations (3) and (4) respectively. New conditions for homeomorphism are derived for eqn. (5) using eventual passivity and monotonicity properties of $\underline{F}$ and $\underline{g}$. In certain results it is sufficient if a P_0 - monotonicity condition is satisfied by $\underline{g}$ i.e., there exists an index j such that $g_j(\underline{x})$ is monotone and a suitable monotonicity condition is satisfied by $\underline{F}$. For eqn. (1), $\underline{f}$ is a homeomorphism if (i) the determinant sign condition is satisfied and (ii) $\underset{\sim}{J}^{e(i)}$, the effective part of the Jacobian of an unbounded region R_i is positive definite or belongs to class P for all i = 1,2, ... , p.

One of the ways to ensure convergence of the solution curve method is to require the initial point to have a unique solution (this is an essential implication of what we refer to as the '1-solution' condition). Several theorems are presented in chapter VII to guarantee the existence of such points for equations of the forms (1), (3), (4) and (5). These conditions make use of certain properties of the piecewise linear functions. In the diagonal case these conditions allow zero slopes (equality sign is allowed in the inequality conditions of $\underline{g}$). Controlled sources are allowed even in the presence of multi-terminal elements. Further, network matrices can be singular. In most of these conditions, we need monotonicity of the piecewise linear functions with respect to a single point $\underline{x}^{(1)}$ only, where $\underline{x}^{(1)}$ is a point every component of which is an arbitrarily large negative number lying in a region with nonsingular Jacobian.

In chapter VIII, new sufficient conditions for homeomorphism are derived based on the C-condition of Prasad [47]. For networks which are an interconnection of two terminal piecewise linear resistors, an (s-2) cofactor condition is presented. This requires checking of the signs of certain cofactors. This condition is further relaxed to unbounded regions. Combinatorial technique [43, 90] is applied to enhance the applicability and reduce the computation in the above results.

Most of the homeomorphism results presented in various chapters have been further generalized making use of a global implicit function theorem. This relaxes the conditions to unbounded regions but requires that certain principal minors of the Jacobians in the unbounded regions have the same sign and are nonzero.

Chapter IX presents the computer implementation of some of the work of this thesis.

Finally in Chapter X, contributions of this thesis are summarised and scope for further research is indicated.

CONTENTS