

SOME NUMERICAL EXPERIMENTS FOR THE  
SIMULATION OF THE ATMOSPHERIC  
AND OCEANIC CIRCULATIONS

By

H.C. UPADHYAYA

Centre for Atmospheric and Fluids Sciences  
Indian Institute of Technology, New Delhi

Submitted to the Indian Institute of Technology, New Delhi  
for the award of the Degree of Doctor of Philosophy

1982

To those from whom I've learned;  
And to my father.

## ACKNOWLEDGEMENTS

The author is sincerely grateful to Professor M.P. SINGH, Head, Centre for Atmospheric and Fluids Sciences (CAFS) and Professor in the Department of Mathematics, IIT, New Delhi, for inspiring guidance and advice during the course of this research. His continual suggestions, constructive criticism and encouragement have been most valuable to me.

I express my profound gratitude to Professor R. SADOURNY, Laboratoire De Meteorologie Dynamique (LMD), Paris, France, for not only his invaluable suggestions and critical review of the progress in research but also for making my stay in France comfortable during the preparation of the first part of the thesis. I am also thankful to Professor A. BERROIR, Director, LMD, Paris, France, for providing all the necessary facilities during my stay in France.

I also express my deep appreciation to Dr. P.K. DAS, Director General for Meteorology, India Meteorological Department, New Delhi, for his interest and advice all along during the course of this study. I am also thankful to Professor R. NATARAJAN, Department of Applied Mechanics, IIT, New Delhi, for fruitful discussions and guidance related to the work contained in Part II of the thesis.

I am no less thankful to Drs. P.C. Sinha, O.P. Sharma, M. Lal, U.C. Mohanty, Padmanabhan, Maithili Saran and Mr. V.M. Sastry, Mr. K.J. Ramesh, Mr. A.D. Rao and other colleagues in the CAFS and Mr. P. Le Van, Mr. G. Rabreau (LMD, Paris) for their helpful suggestions, and all possible assistance during the course of this study.

I am also obliged to the Director, IIT, Delhi, for providing all the necessary facilities for this research work.

I gratefully acknowledge the financial assistance provided to me by the IIT, Delhi, MONEX, Indo-French Collaboration Programme.

I express my thanks to Ms. Neelam Dhody for her expert and accurate typing of the manuscript.

Finally, I would like to thank my wife ANITA for her enduring help and moral support throughout this work.

  
( H.C. UPADHYAYA )

## PREFACE

Various mathematical models in the recent years have been formulated to study certain phenomena in the atmosphere (i.e., frontogenesis and cyclogenesis) and in the ocean (i.e., westward intensification of oceanic gyres) by using different vertical coordinates. Though the normalized pressure coordinate, the sigma system (Phillips, 1957), has provided an efficient treatment of the lower boundary condition, the frontal surfaces resolved much better when modelled in isentropic coordinates (entropy constant). The most attractive feature of the isentropic surfaces is the non-existence of a vertical motion field under adiabatic conditions. This implies that the isentropic surfaces act like material surfaces in the free atmosphere.

This thesis has been divided into two parts. Part I includes mainly the development of a new mathematical model in isentropic coordinate system where we provide an efficient treatment of the lower boundary condition to overcome the difficulties arising due to the intersection of isentropes with the earth's surface. Here we have employed an entropy conserving finite-difference scheme to solve the governing equations of the model atmosphere. In this part of the thesis some numerical simulation experiments and real data (FGGE IIIb, data, 1979) analysis for a hydrostatic, inviscid and

adiabatic atmosphere have been conducted for the atmospheric circulation. The results are discussed for different experiments which differ in their definition of initialization.

Numerical integration experiments with a primitive equation model in isentropic coordinates are also performed to investigate the influence of orography on cyclone formation. Results obtained in the presence of an isolated mountain in different locations in a baroclinic channel atmosphere are illustrated.

Part II deals with a finite element approach to the wind driven ocean circulation study. In the past, the numerical solutions of the wind-driven ocean circulation problems have mainly been achieved by the finite difference or analytical methods. In recent years finite element methods have begun to be used in calculations of fluid dynamics. These methods have the advantage over the finite difference method when the calculations are complicated by the presence of irregular boundaries. For a model barotropic ocean subject to a sinusoidal wind stress, the governing equations contain only two parameters, one a measure of friction and the other is a measure of non-linearity. This system is solved for different relative values of key parameters and the results are discussed here.

---

## CONTENTS

### PREFACE

### PART I

#### CHAPTER I: SOME ATMOSPHERIC PHENOMENA AND THEIR NUMERICAL MODELS

1	INTRODUCTION	2
1.1	FRONTS	4
1.2	FRONTOGENESIS	5
1.3	CYCLOGENESIS	7
1.4	VARIOUS VERTICAL COORDINATE SYSTEMS	12
1.4.1	BASIC EQUATIONS	14
1.4.2	UPPER AND LOWER BOUNDARY CONDITIONS	23
1.5	SOME EARLY WORK ON WEATHER ANALYSIS IN ISENTROPIC COORDINATE SYSTEM	26

#### CHAPTER II: THE NUMERICAL TECHNIQUES

2	INTRODUCTION	41
2.1	THE FINITE DIFFERENCE TECHNIQUE	43
2.1.1	THE ENSTROPY AND ENERGY CONSERVING FINITE DIFFERENCE SCHEMES	45
2.2	FINITE ELEMENT METHOD	49
2.2.1	THE GALERKIN'S METHOD	52
2.3	RELAXATION METHOD	55
2.4	SMOOTHING AND FILTERING	59
2.4.1	THE FILTERING SCHEME	60

CHAPTER III:	NUMERICAL SIMULATION EXPERIMENTS FOR FRONTOGENESIS AND CYCLOGENESIS IN THE ATMOSPHERE	
3	INTRODUCTION	66
3.1	THE PROPOSED MODEL	69
3.1.1	BOUNDARY CONDITIONS	70
3.1.2	INVARIANTS IN THE SYSTEM	71
3.1.3	THE QUASI-GEOSTROPHIC EQUATIONS	74
3.1.4	BOUNDARY CONDITIONS	78
3.2	TREATMENT OF THE LOWER BOUNDARY CONDITION	80
3.3	THE DISCRETIZED FORM OF THE MODEL EQUATIONS	83
3.4	NUMERICAL EXPERIMENTS	85
3.4.1	EXPERIMENT I	85
3.4.2	EXPERIMENT II	88
3.5	REAL DATA ANALYSIS	94
3.5.1	INITIALIZATION OF THE PRIMITIVE EQUATION MODEL	95
3.5.2	THE GOVERNING EQUATIONS IN SPHERICAL COORDINATES ( $\lambda, \phi, \theta$ )	96
3.5.3	NUMERICAL PROCEDURE	98
3.6	TIME-DIFFERENCING SCHEME	99
3.7	RESULTS	100
3.7.1	CONCLUSIONS	105

CHAPTER IV:	INFLUENCE OF OROGRAPHY IN THE LARGE-SCALE MOTION OF THE ATMOSPHERE	
4	INTRODUCTION	138
4.1	DESCRIPTION OF THE MODEL	142
4.2	THE GOVERNING EQUATIONS	144
4.3	RESULTS	150
4.4	CONCLUSION OF THE STUDY PRESENTED IN PART I AND FUTURE CONSIDERATIONS	153

PART II

CHAPTER V: NUMERICAL SIMULATION OF WIND-DRIVEN  
OCEAN CIRCULATION

5	INTRODUCTION	162
5.1	NUMERICAL MODELLING IN OCEAN CIRCULATION	165
5.2	THE GOVERNING EQUATIONS	167
5.3	THE FINITE ELEMENT FORMULATION	169
5.4	NUMERICAL RESULTS	174
5.5	CONCLUSIONS	176
	REFERENCES	182

---