

ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF REINFORCED ROCK SLOPES USING EQUIVALENT MATERIAL CONCEPT

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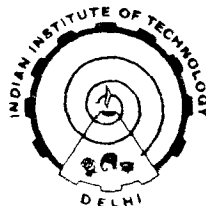
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Thesis submitted

*in fulfilment of the requirements for
the degree of*

DOCTOR OF PHILOSOPHY



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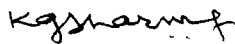
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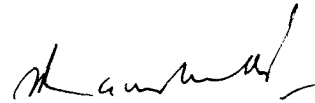
CERTIFICATE

This is to certify that the thesis entitled, "ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF REINFORCED ROCK SLOPES USING EQUIVALENT MATERIAL CONCEPT" being submitted by Mr. A. K. Dhawan to the Indian Institute of Technology, Delhi, for the award of the degree of DOCTOR OF PHILOSOPHY is a record of the bona fide research work carried out by him. Mr. Dhawan has worked under our guidance for the submission of this thesis which to our knowledge has reached the requisite standard.

The thesis or any part thereof has not been submitted to any other University or Institute for the award of any degree or diploma.



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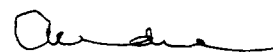
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A.K. DHAWAN

ABSTRACT

Large open cut surface excavations in rock mass are very often necessary both in mining and in civil engineering structures. It is the common objective of the civil and mining engineers to minimize excavation and at the same time to ensure the stability of slopes. Therefore, a rational design procedure is warranted for their safety. The conventional methods of slope stability analysis suffer from one drawback or the other. The Finite Element Method is best suited for the analysis of rock slopes.

The complex behaviour of rock mass is dominated by the presence of fissures, joints and planes of weaknesses. Due to this, the rock slope failure occurs as a result of sliding and/or rotation of blocks or wedges formed by intersecting structural discontinuities. Joint elements can be used in the finite element analysis for modelling a few major discontinuities in the rock, like a fault or crushed seam/layer. Modelling of various types of joints which vary in extent and characteristics is best solved by Equivalent Material Approach and which has been adopted for the present study.

Rock reinforcement has proved to be an effective technique for the improvement of stability of rock slopes. The design of rock reinforcement is usually done by empirical methods. Equivalent Material Approach has been extended to reinforced

jointed rock masses wherein it is possible to determine the optimum orientation, spacing and length of rock reinforcement.

Keeping in view the above facts, a systematic study of the following has been undertaken :

- (i) Formulation of the Equivalent Material Model for reinforced jointed rock mass using Mohr-Coulomb failure criterion for joints and von Mises yield criterion for reinforcement and comparing the response with the available closed form solutions applicable to simple cases of loading (uniaxial compression, uniaxial tension and simple shear),
- (ii) Parametric study to determine the influence of elastic properties and strength parameters of joints and in-situ stress ratio on the factor of safety obtained by using Equivalent Material Approach adopting Mohr-Coulomb failure criterion for a typical 70 m high rock slope with a single set of discontinuities. The important parameters affecting the factor of safety have been identified,
- (iii) Detailed analysis of a slope with a single set of discontinuities and comparison of factors of safety using Equivalent Material Approach, Discrete Joint Approach and Limit Equilibrium Method and studies of improvement in factor of safety by introduction of rock reinforcement.

- (iv) Detailed analysis of slopes to investigate the effect of inclination of joints and angle of slope on the factor of safety for single and multiple sets of discontinuities and to compare the results with Limit Equilibrium Method in the case of single set of discontinuities, and
- (v) Detailed studies to determine factor of safety by introduction of rock reinforcement with varying inclination.

To achieve the above purpose, a small computer program (CPICR) has been developed on NEC-1000-S System first for integration of constitutive relationship using Equivalent Material Approach for reinforced jointed rock mass. To compute plastic strains, Mohr-Coulomb failure criterion has been used for joint/discontinuities and von Mises yield criterion for reinforcement. This program has been verified by comparing the results of sample problems with the closed form solutions. Another computer program (CPRJRM) has been developed on NEC-1000-Ssystem for the elasto-plastic Finite Element Analysis of rock slopes in excavation incorporating the above mentioned constitutive relationships. Eight-noded isoparametric elements have been used and a 2×2 Gauss-Legendre quadrature has been employed for numerical integration. The elasto-viscoplastic algorithm has been used as an artifice to obtain the elasto-plastic solution. The program has also the inbuilt option

to do the analysis either by Equivalent Material Approach or by 6 or 8-noded discrete joint elements.

The present study reveals that the effect of joint spacing, in-situ stress ratio, shear and normal stiffnesses of joints on the factor of safety is insignificant. It is found that factor of safety is affected most by the strength parameters viz. cohesion 'c' and angle of friction, ' ϕ ' and the angle of friction of joint (ϕ) has got the maximum influence on the factor of safety.

It has been observed that the factor of safety of a typical slope with a single set of joints is very close to the factor of safety as calculated by the Discrete Element Approach. This gives the confidence of using the Equivalent Material Approach which considerably reduces the effort required in data preparation and saving in CPU time.

The results of analysis of a typical slope of 70 m height inclined at an angle of 60° containing single set of joints have been presented. The factor of safety has been computed for rock joints inclined at 0° , 15° , 30° , 45° , 75° and 90° . The above studies have been repeated with changes of inclination of slope to 30° , 45° , 75° and 90° . The factor of safety so computed have been compared with those obtained by Limit Equilibrium Method. It has been observed that the Limit Equilibrium Method underestimated the factor of safety in almost all the cases. This shows that considerable economy can be

achieved by employing Finite Element Method of analysis for the design of rock slopes. The effect on improvement of factor of safety by the introduction of rock reinforcement has been studied and it has been found that rock reinforcement enhances the factor of safety of jointed rock slopes considerably. The effect of inclination of reinforcement on factor of safety has also been studied. It has been found that the effect of inclination of reinforcement, when inclined perpendicular to slope face, perpendicular to the joint set or horizontal, on factor of safety is marginal. The factor of safety increases considerably with increase in reinforcement percentage.

The analysis of a typical slope with two sets of orthogonal joint sets has also been carried out. The analysis has been done for three cases consisting of two sets of orthogonal joint sets at varying inclinations. It has again been observed that the introduction of rock reinforcement greatly enhances the factor of safety. The studies of change of inclination of reinforcement on the stability of rock slopes shows that the effect of inclination of reinforcement when inclined either perpendicular to slope face or perpendicular to critical joint set or horizontal, on factor of safety is marginal.

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