

A THESIS ON
GENERATING FUNCTIONS AND OTHER RESULTS FOR CERTAIN
POLYNOMIALS INVOLVING TWO OR MORE VARIABLES

By
A. Bhaskar Rao
Department of Mathematics
Indian Institute of Technology
New Delhi

Submitted to the Indian Institute of
Technology, New Delhi for the award
of the Degree of Doctor of Philosophy
in Mathematics
1973

C E R T I F I C A T E

This is to certify that the thesis entitled 'Generating Functions And Other Results For Certain Polynomials Involving Two or More Variables' which is being submitted by Mr. A. Bhaskar Rao for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

H. L. Manocha

(H.L. Manocha)
Assistant Professor
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110029

A C K N O W L E D G E M E N T S

It gives me a great pleasure to express my regards and a profound sense of gratitude to Dr. H.L. Manocha, M.A., Ph.D., Assistant Professor, Department of Mathematics, Indian Institute of Technology, New Delhi, for his kind supervision, valuable guidance and constant help throughout the preparation of this thesis. But for his keen interest in my work it would not have been possible to complete the work.

I am thankful to Professor M.K. Jain, M.A., D.Phil., D.Sc., the Head of the Department of Mathematics, Indian Institute of Technology, New Delhi, for his keen interest in my work.

I will be certainly failing in my duty if I do not thank Dr. (Mrs.) Aruna Srivastava for her suggestions and help in the preparation of the thesis.

My thanks are also due to the authorities of the Indian Institute of Technology, Delhi for providing me with a scholarship and all the facilities of the institute during my research work.

I am very much thankful to Mr. R.K.S. Rathore and Mr. Rajendra Prakash for giving me their valuable time in reading the manuscript, making detailed corrections and many helpful suggestions.

I finally, thank Mr. D.R. Joshi for his commendable work in typing the manuscript.

Bhaskar Rao A
(Bhaskar Rao, A.)

Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110029.

C O N T E N T S

Chapter		Page
0	INTRODUCTION	I - XVIII
	0.1 Generalized Hypergeometric Functions	II
	0.2 Orthogonal Polynomials	XI
	0.3 Brief Summary of the Thesis	XIV
	0.4 References	XVII
I	ON A POLYNOMIAL OF THE FORM F_4	1 - 36
	1 Definition and Generating Function	2
	2 Relationships Involving Jacobi Polynomials	7
	3 Recurrence Relation	10
	4 Special Properties	12
	5 Bilinear Generating Functions	14
	6 Integral Representation	21
	7 A Theorem Connected with the f Polynomial	24
	References	36
II	ON A POLYNOMIAL OF THE FORM F_D	37 - 59
	1 Definition and Generating Functions	38
	2 Recurrence Relations	43
	3 Expansion and Special Properties	48

	4	Another Definition for S_n	52
	5	Other Generating Functions	54
		References	59
III		ON A POLYNOMIAL OF THE FORM F_A	60 - 82
	1	Definition and Generating Function	61
	2	Special Properties	63
	3	Other Generating Functions	70
	4	Recurrence Relation	74
	5	Bilinear Generating Relation	76
		Reference	82
IV		OPERATIONAL FORMULAE CONNECTED WITH THE TWO GENERALIZATIONS OF GEGENBAUER POLYNOMIALS	83 - 98
	1	Introduction	84
	2	Generalization of (1.1)	84
	3	Generating Function	87
	4	An Extension to (2.7) and (2.8)	90
	5	Extension to Gegenbauer Polynomial	91
	6	Relations Involving the Operator (5.1)	93
		Reference	98
V		ON A THEOREM BY BROWN AND CHRISTOFFEL DARBOUX FORMULA	99 - 113
	1	Introduction	100
	2	Generalization of the Theorem by Brown	102

	3	Summation Formula for the Lauricella's Function F_A	108
		References	113
VI		EXPANSION FORMULAE FOR LOMMEL'S FUNCTION	114 - 125
	1	Introduction	115
	2	Outline of the Method	115
	3	Raising and Lowering Operators for the index ν	118
	4	Expansion Formulae	120
		Reference	125
VII		INTEGRAL EXPRESSIONS AND GENERATING FUNCTIONS BY MEANS OF FRACTIONAL DERIVATIVES	126 - 174
	1	Introduction	128
	2	Rules for Fractional Integration and Differentiation	130
	3	Theorem on Term by Term Fractional Differentiation (Integration)	131
	4	Derivations of (1.3) and (1.4)	133
	5	Transformations of (1.3) and (1.4) By Fractional Integration by parts	137
	6	Generating Functions by Fractional Derivatives	148
	7	Generating Relations Involving Jacobi Polynomials	157

	8	Generating Functions Involving Lauricella's Functions F_A	162
	9	Convergence Conditions	168
		References	173
VIII		A THEOREM ON FRACTIONAL DERIVATIVES AND ITS APPLICATIONS	175 - 210
	1	A Theorem on Fractional Derivatives	176
	2	Some Elementary Results by Means of Fractional Derivatives	180
	3	Application of the Theorem (1.1)	185
	4	Formulae Involving Trigonometric Functions	200
		References	210