

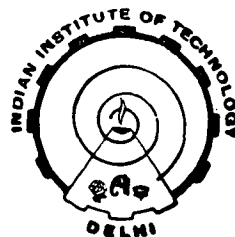
DESIGN OF MAXIMALLY-FLAT AND MONOTONIC FIR FILTERS USING THE BERNSTEIN POLYNOMIAL

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November, 1987

to my
parents

CERTIFICATE

This is to certify that the thesis entitled, "Design of Maximally-Flat and Monotonic FIR Filters using the Bernstein Polynomial" being submitted by L.R.Rajagopal to the Department of Electrical Engineering, Indian Institute of Technology, New Delhi, for the award of the degree of Doctor of Philosophy, is a record of bona fide research work carried out by him under our supervision and guidance and in our opinion, it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.



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ABSTRACT

Linear-phase FIR filters find wide applications where frequency dispersion due to non-linear phase is harmful, as in speech processing and data transmission. Presently, a variety of design procedures are available for the design of this class of filters and most of them result in ripples in the passband as well as the stopband. However, there are some applications, as in the design of filter banks, where we desire monotonic frequency response to overcome the problem of inter-channel interference. Maximally-Flat (MAXFLAT) FIR filters, characterized by flat response in both passband and stopband, are useful in such applications. A number of methods exist for designing MAXFLAT FIR filters; none of them, however, gives the order of the filter and the order of tangency at $\omega = 0$ or $\omega = \pi$ for arbitrary magnitude specifications and this deficiency has restricted the use of this class of filters. Besides, although a number of direct and indirect methods are available for computing the coefficients of MAXFLAT FIR filters, all of them are involved and require a large dynamic range of the computing machine.

The present research work introduces a new technique for the design of MAXFLAT FIR filters by using the Bernstein polynomial, through which an equivalence is established between the earlier known methods. It is shown that the design of MAXFLAT FIR

filters through this new approach gives an additional insight into the physical significance of the order of flatness. Extending this concept of design, a matrix approach is proposed for determining the coefficients of MAXFLAT FIR filters efficiently. The limitations of dynamic range and computational complexities of the earlier methods are overcome by this new method.

A significant contribution of this thesis is the design procedure proposed for designing MAXFLAT FIR filters with arbitrary magnitude specifications. Using a set of recurrence relations, this method simultaneously searches for the optimal order of filter and the degrees of tangency at $\omega = 0$ and $\omega = \pi$. Also, being an optimal method, it results in the minimum order of the filter required to meet the given specifications and a detailed comparison with the earlier methods establishes the superiority of this approach.

Generally, in the filter design problem, we are interested in meeting the passband edge specifications exactly and usually, the order of the MAXFLAT FIR filter required to meet this criteria is very high. One of the solutions to this problem is to design sub-optimal filters with monotonic frequency response which meet the requirement of passband edge exactly. The sub-optimal filters are designed by considering a linear combination of two or three MAXFLAT filters. Here again, we establish a relationship between the earlier methods and the present method. We also show that, for a set of given specifications, this class

of filters yields considerable saving in the required order of the filter as compared to MAXFLAT filters.

The theory of MAXFLAT FIR filters developed here can be used to design other classes of filters like the Quadrature Mirror filters (QMF), multi-band filters and two-dimensional MAXFLAT FIR filters. In this thesis, we study the first case in details and propose an efficient method for designing MAXFLAT QMFs. We extend the design of monotonic filters to generate QMFs with low reconstruction errors. Finally, a design tool in FORTRAN is provided to design MAXFLAT and monotonic FIR filters.

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