ANALYSIS OF FLAT AND CURVED PANELS USING ELEMENT FREE GALERKIN METHOD

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by

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Certificate

This is to certify that the thesis entitled, **'Analysis of Flat and Curved Panels using Element Free Galerkin Method'** submitted by **Mr. Gaurav Watts** to the **Indian Institute of Technology Delhi** for the award of degree of **Doctor of Philosophy** embodies original research work done by him under our supervision. In our opinion, the thesis work meets the requisite standards and the candidate is worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the institute. The contents of this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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Place: New Delhi, India

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Gaurav Watts

Abstract

The present work deals with the nonlinear bending and stability behaviours of thin and moderately thick, flat and curved panels. The first order shear deformation theory in combination with Marguerre-von Kármán strain-displacement assumptions is employed to model the flat plates and shallow shell panels on rectangular planform, while Sanders' shell theory is used to analyze shallow conical shell panels. Element free Galerkin (EFG) method with moving kriging (MK) shape functions is employed to transform the governing integral weak form into a set of algebraic equations. An in-house MATLAB code is developed to solve the linear/nonlinear equations of equilibrium for flat and curved panels. Nonlinear equilibrium paths are obtained using the modified Riks technique in conjunction with Newton-Raphson method.

At the beginning, the efficiency of the present numerical technique (EFG with MK shape function along with the relevance / effectiveness of associated parameters, e.g., the support domain) in dealing with the "shear locking problem of Reissner-Mindlin plates", and "membrane locking problem of curved panels" is examined by a series of convergence studies and comparison of the present numerical results with available analytical or numerical solutions. Thereafter, several new problems on bending, buckling and vibration behaviours of thin structural panels made up of isotropic and laminated composite materials are taken up for investigation, with particular focus to non-rectangular geometry, for which research publications are limited. An attempt is made to provide the numerical results on the "bending and buckling of structural panels" in the "non-dimensional form", so that they can be easily used as "benchmark solutions" by engineers and researchers in the future.

First, the flexural (bending and buckling) behaviours of various flat panels (rectangular, skew, trapezoidal and arbitrary straight-sided quadrilateral) under a variety of practical loading and boundary conditions (edge supported, corner supported, continuous over multiple spans, supported on elastic foundation) are examined.

Next, the nonlinear bending and stability characteristics of various curved shell panels with rectangular planform (cylindrical, spherical, hypar and conoidal) and non-rectangular planform (conical) are studied for different geometrical parameters (circumferential angle, radius-to-span ratio, span-to thickness ratio, semi-vertex angle, etc.) and boundary conditions. Attempts are made to obtain the representative snap-through responses (hardening to softening) and possible equilibrium configurations of various curved panels under different cases of symmetric and un-symmetric bending.

Finally, the present numerical technique (EFG method with MK shape function) is extended for the free flexural vibration analysis of flat panels in contact with bounded fluid. The fluid is assumed to be ideal and the effect of free surface waves is neglected. Potential flow theory is assumed and the velocity potential is used to calculate the added mass effect due to the fluid. The effect of various structural and fluid domain parameters on natural frequencies of thin isotropic and laminated composite skew/trapezoidal bottom of a rectangular tank filled with fluid is investigated in detail.

सार

वर्तमान कार्य पतली और मध्यम मोटी, समतल और घुमावदार पैनलों के गैर रेखीय वंकन और स्थिरता व्यवहार से संबंधित है। मार्गुरे–वॉन कर्मण तनाव विस्थापन धारणाओं के साथ संयोजन में फर्स्ट आर्डर शियर ड़ेफोरमेशन थ्योरी को आयातकार प्लानफोर्म पर समतल प्लेट्स और उथले खोल पैनलों को मॉडल करने के लिए नियोजित किया गया है, जबकि सैंडर्स के खोल सिद्धांत का उपयोग उथले शंकु खोल पैनलों का विश्लेषण करने के लिए किया गया है। मूविंग क्रिगिंग (एमके) शेप फंक्शन के साथ एलिमेंट फ्री गैलेरकिन (ईएफजी) विधि गवर्निंग इंटीग्रल वीक फॉर्म को बीजगणितीय समीकरणों के सेट में बदलने के लिए नियोजित की गयी है। समतल और घुमावदार पैनलों के रैखिक / गैररेखीय समीकरणों को हल करने के लिए एक मैटलैब कोड विकसित किया गया है। न्यूटन–रैपसन विधि के संयोजन के साथ संशोधित रिक्स तकनीक का उपयोग करके गैररेखीय समतल पथ प्राप्त किए गए हैं।

शुरुआत में, वर्तमान संख्यात्मक तकनीक की दक्षता (इफजी तकनीक के साथ एमके शेप फंक्शन के साथ संबंधित पैरामीटर की प्रासंगिकता / प्रभावशीलता, उदाहरण के लिए, सपोर्ट डोमेन) रिसीनर–मिंडलिन प्लेट्स की शीयर लॉकिंग समस्या से निपटने में, और घुमावदार पैनलों की मेम्ब्रेन लॉकिंग समस्या अभिसरण अध्ययन की एक श्रृंखला और वर्तमान संख्यात्मक परिणामों की तुलना उपलब्ध विश्लेषणात्मक या संख्यात्मक समाधान के साथ की गयी है। इसके बाद इसोट्रोपिक और लैमिनेटेड कम्पोजिट से बने पतले पैनलों के झुकाव, बकलिंग और कंपन व्यवहार पर कई नई समस्याएं जांच के लिए ली गयी हैं, विशेष रूप से गैर–आयताकार जेओमैट्री पर ध्यान केंद्रित हैं, जिसके लिए शोध प्रकाशन सीमित हैं। 'गैर–आयामी रूप' में 'झुकाव और संरचनात्मक पैनलों की बकलिंग' पर संख्यात्मक परिणाम प्रदान करने का प्रयास किया गया है, ताकि भविष्य में इंजीनियरों और शोधकर्ताओं द्वारा उन्हें आसानी से बेंचमार्क समाधान के रूप में उपयोग किया जा सके।

सबसे पहले, विभिन्न समतल पैनलों (आयताकार, तिरछा, समलम्ब और सीधे पक्षीय चतुर्भुज) के विभिन्न प्रकार के व्यावहारिक लोडिंग और सीमा परिस्थितियों के तहत (बेंडिंग और बक्लिंग) व्यवहार (किनारे समर्थित, कोने समर्थित, निरंतर एकाधिक स्पैन, लोचदार नींव पर समर्थित) की जांच की गयी है।

इसके पश्चात, आयताकार प्लानफोर्म (बेलनाकार, गोलाकार, हाइपर और कनोईडल) और गैर आयताकार प्लानफोर्म (शंकु) के साथ विभिन्न घुमावदार खोल पैनलों के गैररेखीय झुकने और स्थिरता विशेषताओं का अध्ययन विभिन्न ज्यामितीय पैरामीटर (परिधीय कोण, त्रिज्याअवधि अनुपात, अवधि–मोटाई अनुपात, अद्धर्–चरम कोण, आदि) के और सीमा की स्थिति लिए किया गया है । स्नैप–थ्रू प्रतिक्रियाओं और सममित और गैर–सममित झुकने के विभिन्न मामलों के तहत विभिन्न घुमावदार पैनलों के संभावित समतोल विन्यास को प्राप्त करने के प्रयास किए गए है ।

अंत में, वर्तमान संख्यात्मक तकनीक (एमके शेप फंक्शन के साथ ईएफजी विधि) बाध्य तरल पदार्थ के संपर्क में समतल पैनलों के मुक्त कंपन विश्लेषण के लिए इस्तेमाल किया गया है। तरल पदार्थ आदर्श माना गया है और मुक्त सतह तरंगों का प्रभाव उपेक्षित है। पोटेंशियल प्रवाह सिद्धांत माना गया है और तरल पदार्थ के कारण द्रव्यमान प्रभाव प्राप्त करने के लिए वेग क्षमता का उपयोग किया गया है। तरल पदार्थ से भरे आयताकार टैंक के पतले इसोट्रोपिक और लैमिनेटेड कम्पोजिट तिरछे / समलम्बाकार तल की प्राकृतिक आवृत्तियों पर विभिन्न संरचनात्मक और द्रव डोमेन पैरामीटर का प्रभाव विस्तार से जांच किया गया है।

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Greek Symbols

α	Semi vertex angle of the conical shell panel, see equation (4.42), page 37
β_f	Angle made by x_1 axis of lamina with x axis of principle material coordinates in counterclockwise sense, see equation (4.25), page 33
ν	Poisson's Ratio, see equation (4.21), page 31
ω	Natural frequency of the structure, see equation (4.90), page 47
ω^f	Natural frequency of the structure in contact with fluid, see equation (4.90), page 47
ω^v	Natural frequency of the structure in dry environment , see equation (4.90), page 47
ϕ	Velocity potential function, see equation (4.52), page 40
ψ	Circumferential substended angle of the shallow shell panels or Skew angle for the skew plate, see equation (4.42), page 37
ρ_s	Density of the structural material, see equation (4.66), page 43
σ	Stress field in the structure, see equation (4.66), page 43
θ_x	Rotation of mid-plane about y-axis , see equation (4.28), page 33
θ_y	Rotation of mid-plane about x-axis , see equation (4.28), page 33
ε	Strain field in the structure, see equation (4.66), page 43
ε_b	Bending strains, see equation (4.31), page 34
ε_m^L	Linear component of membrane strains, see equation (4.31), page 34

ε_m^{NL} Nonlinear component of membrane strains, see equation (4.31), page 34

ε_s Shear strains, see equation (4.31), page 34

φ Meshfree Shape Function, see equation (4.13), page 28

ξ, η, ζ Orthogonal coordinate system of the rectangular container considered for fluid-structure interaction problems, see equation (4.52), page 40

Γ_{p} Surface Area of the Structure, see equation (4.66), page 43

Ω_{p} Bounded volume of the Structure, see equation (4.66), page 43

Other Symbols

$\bar{\mathbf{b}}$ Body Force on the Structure, see equation (4.66), page 43

$\bar{\mathbf{t}}$ Traction Force on the Structure, see equation (4.66), page 43

$\mathbf{M_f}$ Added mass matrix due to the fluid, see equation (4.66), page 43

$\mathbf{F}$ Force Vector, see equation (4.75), page 45

$\mathbf{K_s}$ Structural Stiffness Matrix, see equation (4.75), page 45

$\mathbf{K_T}$ Tangent Stiffness Matrix, see equation (4.75), page 45

$\mathbf{K_W}$ Added stiffness matrix due to Winkler Foundation, see equation (4.75), page 45

$\mathbf{M_s}$ Structural Mass Matrix, see equation (4.92), page 47

a Length of the structure, see equation (4.66), page 43

b Width of the structure, see equation (4.66), page 43

cor Correlation function in moving kriging shape functions , see equation (4.15), page 29

d_s Size of Support Domain, page 30

E Young's modulus of the structual material, see equation (4.21), page 31

G	Shear Modulus, see equation (4.21), page 31
h	Thickness of the structure, see equation (4.66), page 43
q	Load acting on the Structure, see equation (4.83), page 46
R	Radius of the shallow shell panels, see equation (4.42), page 37
R_1	Shorter radius of the conical panel, see equation (4.42), page 37
R_2	Larger radius of the conical panel, see equation (4.42), page 37
u_0	In-plane displacement of a point at midplane in x direction, see equation (4.28), page 33
v_0	In-plane displacement of a point at midplane in y direction, see equation (4.28), page 33
w_0	Transverse displacement of a point at midplane , see equation (4.28), page 33
w_1	Initial stress free profile of shallow shell , see equation (4.28), page 33
x, y, z	Orthogonal coordinate system of the structure, see equation (4.40), page 36
k_w	Elastic subgrade modulus of the foundation, see equation (4.79), page 45
$\mathbf{d}$	Displacement Vector, see equation (4.75), page 45

Acronyms

BEM	Boundary Element Method
DQM	Differential Quadrature Method
EFG	Element Free Galerkin
FDM	Finite Difference Method
FEM	Finite Element Method
FGM	Functionally Graded Material
FSDT	First Order Shear Deformation Theory
GDQM	Generalized Differential Quadrature Method
HSDT	Higher Order Shear Deformation Theory
MLS	Moving Least Squares
MK	Moving Kriging
PIM	Point Interpolation Method
RKPM	Reproducing Kernel Particle Method
RPIM	Radial Point Interpolation Method
SPH	Smooth Particle Hydrodynamics
SSL	Sinusoidal Load
UDL	Uniformly Distributed Load