GENERATING RELATIONS AND DIFFERENTIAL OPERATOR REPRESENTATIONS

by

ASHOK KUMAR AGARWAL

Department of Mathematics
Indian Institute of Technology, Delhi

Submitted
in fulfilment of the requirements of the degree of
DOCTOR OF PHILOSOPHY
to the
Indian Institute of Technology, Delhi

January, 1980

CERTIFICATE

This is to certify that the thesis entitled "GENERATING RELATIONS AND DIFFERENTIAL OPERATOR REPRESENTATIONS" being submitted by Shri Ashok Kumar Agarwal to the Indian Institute of Technology, Delhi for the award of Doctor of Philosophy in Mathematics is a record of bonafide research work carried out by him. Shri Ashok Kumar Agarwal has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis, which to the best of my knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

(H.L. Manocha)
Associate Professor
Department of Mathematics
Indian Institute of Technolog
Hauz Khas, New Delhi-110029
India

ACKNOWLEDGMENTS

I am deeply indebted to Dr. H.L. Manocha, Associate Professor, Department of Mathematics, Indian Institute of Technology, Delhi, who has been a source of continuous inspiration, for his advice, guidance and patient supervision of my present work.

I am thankful to Professor O.P. Bhutani, Head of the Department of Mathematics, Indian Institute of Technology, Delhi for his constant encouragement and help throughout this work.

My thanks are due to the Director, Indian Institute of Technology, Delhi for having placed at my disposal all the facilities necessary for my research work.

I also take this opportunity to thank Dr. Roshan Singh, Principal, R.B.S. College, Agra and Dr. Har Dutt Singh, Head of the Department of Mathematics, R.B.S. College, Agra for their cooperation and everwilling help.

I owe, at large, to the nation which bears the social cost of what I received through the F.I.P. fellowship for my research work.

My thanks are also due to Mr. Dev Raj Joshi for his painstaking typing.

AK Agarwal

(Ashok Kumar Agarwal)

SYNOPSIS

This thesis consists of eight chapters. Chapters II, III, VI and VII embody the bulk of the working. Chapterwise details are as follows:

Chapter I: INTRODUCTION

The purpose of this chapter is to provide a survey of special functions occuring in the present work. A brief historical review of the previous work done on this subject is also given.

Chapter II: EXTENSIONS OF BILATERAL GENERATING RELATIONS

This chapter is devoted to the study of the generating functions of the type

$$\sum_{n=0}^{\infty} A_{m,n} S_{m+n}(x) t^n = \frac{f(x,t)}{[g(x,t)]^m} S_m(h(x,t)),$$

for the sequence of functions $\{S_n(x) | n = 0,1,2,\ldots\}$, where m is a non-negative integer, the $A_{m,n}$ are arbitrary constants, and f,g,h are arbitrary functions of x and t. In fact, this type of the generating functions provides us with the basic tool for obtaining bilateral generating functions. Many results due to Srivastava, Chaterjea, Mehler, Hardy-Hille, Buchholz, Jain and Manocha, Weisner, Manocha, Srivastava and Singhal have been either extended or generalized. New generating functions for the Gottlieb, Meixner, Cesaropolynomials and the generalized Sylvester polynomials are also obtained.

Chapter III: GENERATING FUNCTIONS BY AN IDENTITY

Making use of the Lagrange's expansion formula, we, in this chapter, establish an identity which helps us in obtaining new generating relations for both single as well as multivariable functions. The particular cases of these generating functions are seen scattered in the literature.

Chapter IV: GENERATING FUNCTIONS BY AN OPERATIONAL FORMULA

In this chapter we generalize some known generating relations by using the operator $T_\mu = t(\mu + td/dt)$ due to Mittal. It is also shown that these results can be further generalized by using the fractional derivative concept.

Chapter V: THE UNIFIED PRESENTATION OF OPERATIONAL FORMULAE AND GENERATING FUNCTIONS FOR CERTAIN SPECIAL FUNCTIONS

In this chapter an attempt is made to present an elegant generalization of operational formulae and generating functions which have been obtained by Srivastava and Singhal for generalized polynomials $\{T_n^{(\alpha,\beta)}(x,a,b,c,d,p,r)\}$. The results obtained in this chapter include linear and bilateral generating functions and operational formulae for the polynomials $\{S_n^{(\alpha,\beta)}(x,a,b,c,d,e,f,p,r)\,|\,n = 0,1,2,\ldots\}$ defined by the Rodrigue's type formula

$$S_n^{(\alpha,\beta)}(x,a,b,c,d,e,f,p,r) = \frac{(ax^e+b)^{-\alpha}(cx^f+d)^{-\beta}}{n!}\exp(px^r)$$

$$.D^n\{(ax^e+b)^{n+\alpha}(cx^f+d)^{n+\beta}\exp(-px^r)\}.$$

Some formulae due to Patil and Thakare and Manocha and Sharma are deduced as particular cases.

Chapter VI : OPERATIONAL GENERATING FORMULAE FOR CERTAIN FUNCTIONS INVOLVING SEVERAL VARIABLES

This chapter is devoted to introducing two multivariable extensions of the operator formula ${}_aT_b = Z^a(b+a\, d/dz)$ due to Mittal. This leads to, among others, the generalization of his basic results.

Chapter VII : ALTERNATIVE METHODS OF OBTAINING DIFFERENTIAL OPERATOR REPRESENTATIONS FOR CERTAIN SPECIAL FUNCTIONS

Representation for the Jackson-Hermite polynomials in terms of a differential operator involving their generating function was obtained by Poli and for the Gegenbauer polynomials by Haradze Similar representations for the polynomials of Laguerre, Meixner and Poisson-Charlier were obtained recently by Allaway. In this chapter, we derive such representations by alternative methods. Results obtained here include, in addition to the Jackson-Hermite, Gegenbauer, Laguerre, Meixner and Poisson-Charlier polynomials, the Hermite, modified Jacobi, generalized Sylvester, Bessel, Konhauser biorthogonal and Srivastava and Singhal polynomials.

Chapter VIII : REMARKS ON THE ZEROS OF CERTAIN CLASSICAL POLYNOMIALS

In this chapter the zeros of certain classical polynomials have been identified with the eigen values of various matrices.

CONTENTS