ON PROPAGATION OF SONIC AND SHOCK WAVES IN PHYSICAL GAS DYNAMICS

BY

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...TO MY PARENTS...

CERTIFICATE

This is to certify that the thesis entitled 'ON PROPAGATION OF SONIC AND SHOCK WAVES IN PHYSICAL GASDYNAMICS' which is being submitted by Mr. S. Sitharama Prabhu for the award of degree of Doctor of Philosophy in Mathematics to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

This thesis entitled 'ON PROPAGATION OF SONIC AND SHOCK WAVES IN PHYSICAL GASDYNAMICS' consists of five chapters. The first chapter gives a brief historical review of the previous studies related to the present work and a discussion concerning the set of equations governing the gas flows under study.

In the second chapter we have studied the propagation of weak discontinuities in thermally conducting and radiating gas flows, relaxing and radiating gas flows and radiation magneto gasdynamic flows. More specifically, using singular surface theory, the differential equations governing the growth and decay of these discontinuities in uniform and non-uniform propagation have been formulated and solved completely. Further the criteria for blowing up of these discontinuities have been established. It turns out that in uniform propagation, the coupling of radiation and relaxation has a stabilizing effect and thus increases the time taken to form a shock. For non-uniform propagation the effect of radiation and magnetic field is to delay the development of a shock while the electrical conductivity causes the rapid onset of it.

The method of characteristics has been employed in Chapter-III to study the propagation of acceleration waves

in non-equilibrium gas flows, non-equilibrium magneto gas-dynamic flows and radiation magneto gasdynamic flows. The differential equations governing the local and global behaviour of the amplitude of the wave are derived and solved. The critical value of the initial amplitude and critical time for breakdown of the acceleration wave at the cusp of the envelope of the intersecting forward characteristics have been obtained. It turns out that the non-equilibrium character of the gas accelerates the process of flattening of the wave fronts and hence causes the shock formation time to increase. Similarly the coupling of radiation and magnetic field exhibits a stabilizing effect and increases the time for the formation of a cylindrical shock.

In Chapter-IV we have studied the geometry of radiative and thermally conducting gas flows. More specifically, we have obtained the expressions for curvature and torsion of the streamlines in terms of velocity components, pressure, density, thermal conductivity and radiation variables. This is followed by the determination of pressure gradient along the streamlines, their principal normals and binormals. It has been shown that the radiating character of the gas decreases the pressure gradient whereas thermal conductivity increases it along the streamlines but the pressure remains constant along the binormals and if the streamlines are straight lines, the trajectories of the principal normals also lie on the surface of the constant pressure. Finally, the expressions for vorticity components in terms of curvature of the streamlines and the velocity gradients along the streamlines, their principal normals and binormals have been obtained.

Under the assumption that the shock configuration is a continuous differentiable function of coordinates, we have studied in Chapter V, the following topics:

(i) gradients of flow variables behind a shock

(ii) components of vorticity

(iii) curvature of an attached shock in radiative and vibrational non-equilibrium gasdynamics.

The complete chapter has been divided into two sections i.e., section A and section B. In section A, we have studied the above problems using Taub's analysis whereas in section B we have carried over the aforementioned analysis for pseudo-stationary shocks in radiative and vibrationally relaxing gas flows. It turns out that in the absence of vibrational relaxation,the ideal radiating character of the gas increases the velocity gradient and decreases the pressure gradient across the shock. Further assuming an oblique shock wave attached to the leading edge of the obstacle appears from the vertex so that the flow after the shock is along the surface of the obstacle,we have shown that the attached shock wave at the vertex in two dimensional wedge becomes curved due to the coupling of radiation and vibrational relaxation. As the vorticity is independent of energy equation, in such flows, it is unaffected and remains the same as in the pseudo-stationary flows.

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