MEAN GRADIENT DESCENTA
NEW OPTIMIZATION APPROACH WITH APPLICATIONS TO INVERSE PROBLEMS IN IMAGING

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DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
JUNE 2023

Mean Gradient Descent-

A New Optimization Approach with Applications to Inverse Problems in Imaging

by

Sunaina

Department of Physics

submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the

INDIAN INSTITUTE OF TECHNOLOGY DELHI

JUNE 2023

Dedicated to my teachers and family

CERTIFICATE

This is to certify that the thesis entitled, "**Mean gradient descent- A New Optimization Approach with Applications to Inverse Problems in Imaging**", being submitted by **Sunaina**, to the Indian Institute of Technology Delhi, for the award of degree of **Doctor of Philosophy** in the Department of Physics is a record of bonafide research work carried out by her under our supervision and guidance. She has fulfilled the requirements for the submission of the thesis, which to the best of our knowledge has reached the required standard.

The material contained in the thesis has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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ACKNOWLEDGEMENTS

I wish to express my heartfelt gratitude to my PhD advisor **Prof. Kedar Khare** for his unwavering support, exceptional guidance and his devoted effort throughout my PhD journey. His critical feedback, deep insights, and encouragement have been invaluable in shaping my ideas and producing a high-quality thesis. His astute and perceptive approach to research was crucial in successfully completing this thesis. I deeply appreciate the valuable lessons he imparted specifically on science ethics and human values. His unwavering positivity, exceptional guidance and kind encouragement has helped me to evolve both as a researcher and as a person. It is a great privilege to be his student.

I would also like to thank my committee members **Prof. P. Senthilkumaran**, **Prof. Aloka Sinha**, **Prof. Manideepa Banerjee**, and **Prof. Kedar khare**, for their insightful feedback and constructive criticism. Their comments and suggestions have helped me to refine my work and make it more impactful. I am immensely grateful to **Prof. P. Senthilkumaran** and **Prof. Joby Joseph** for providing me with interesting and engaging lectures in Optics.

I would like to express my profound gratitude to the teachers and professors during my school and graduate studies who played a crucial role in shaping my formative years. I am grateful to my Mathematics teacher in Guru Nank International School, Ludhiana: **Mr. Naveen Bansal** whose interesting lessons have changed the trajectory of my scientific career. I express my deepest gratitude to **Prof. Mohammad Sajjad Athar** who has been a constant source of inspiration all through my life. I am thankful to **Prof. Abbas Ali**, **Prof. Mohammad Shoeb** for their wonderful lectures in my graduate studies from Aligarh Muslim University.

I am thankful to the Indian Institution of Technology Delhi for providing me with a five-year institute assistantship and financial assistance to participate in national and international conferences. Additionally, I appreciate the efforts of the IIT Delhi hostel staff for taking care of the hostel residents in all possible aspects.

I would like to acknowledge my colleagues **Priyanka Lohchab**, **Apoorv Pant**, **Mansi Butola**, **Ritika Malik**, for all the interesting discussions. I am thankful to **Mansi Butola** and **Jasleen Birdi** for being a wonderful collaborators.

I would also like to appreciate my team members of IIT Delhi student chapter of Optica, **Anisha Pathak**, **Shashank Gahlaut**, **Nabarun Saha**, **Sugeet Sunder**, **Sub-**

hajit Karmakar, **Kalpak Gupta**, **Vikas Kumar**, **Vivek Semwal**, **Hemant Kumar Singh**, **Omshankar**, **Akanksha Angural** and **Sukhwinder**; who have been the part of exciting activities including science outreaches, optics and photonics seminars, IONS 2020 conference organized at IIT Delhi and several others.

I am incredibly grateful to my dear friends, **Mansi Butola**, **Hemant Kumar Singh**, **Shilpa Sharma**, **Milton Mondal**, **Pragati Sharma**, **Preeti**, and **Anisha Pathak**, for their invaluable contribution to this journey, which has been both enriching and enjoyable. I consider them all as a real treasure in my life. I am grateful to my family for their love and support. My mother's (**Smt. Meena Kumari**) lovely smiles and father's (**Sh. Gajendra Singh**) faith in me has been a constant source of encouragement throughout my PhD years. It is due to the immense love and blessings of my grandfather, **Sh. Daryao Singh**, that I am where I am today. My siblings, **Vishal Kumar** and **Anjali**, are not only my most significant critics but also my source of motivation and support.

I extend my deepest gratitude to **Haimanthi Mukhopadhyay** for introducing me to the real essence of Yoga in life. Undoubtedly, the practice of yoga has contributed to making my PhD years a smoother experience. I am indebted for the precious teachings and practices which will accompany for my whole life.

Once again, I express my sincere appreciation to everyone who have devoted their precious time and energy for making my PhD years worthwhile.

Sunaina

ABSTRACT

Modern imaging systems are the synergic meld of unconventional hardware design and sophisticated algorithms. The typical data in these systems do not even visually resemble the image of an object whereas it is in a coded form. The task of image reconstruction algorithms is to invert this coding by solving a mathematical problem and provide a visually interpretable image. The major challenge in solving the inverse imaging problem is that the acquired data is incomplete and noisy. An efficient framework to solve such problems involves optimization of specific objectives like data-fit error and sparsity penalty by minimizing a combined cost function. This formulation offers a great possibility to achieve unconventional image reconstruction which is otherwise not possible with traditional imaging techniques. However, a basic roadblock preventing the application of optimization algorithms in practical imaging devices is the involvement of a free parameter called regularization parameter. The quality of the recovered object is critically dependent on its choice. Moreover, the tuning procedure involves subjective intervention for image quality assessment. The daunting task of tuning the free parameter can initimidate the end-users to possibly switch to naive imaging techniques with sub-optimal performance. The thesis addresses this problem by proposing a novel optimization framework of 'mean gradient descent (MGD)' which does not involve such critical free parameters. The idea of MGD is based on an interesting perspective of attaining a balance between the data-fit error and regularization terms instead of attaining a total cost minimization. The new methodology is simple to implement computationally and provides a generic framework to solve a range of inverse problems in imaging. The proposed framework is observed to successfully solve the important inverse imaging problems of image deconvolution, quantitative phase imaging and the problem of 3D imaging in digital holography. With the robust and adaptive framework of MGD, it is possible to get phase recovery with full detector resolution and accuracy better than shot-noise limit from a single-shot hologram data. The thesis further delves into intriguing and fundamental inquiries concerning the concept of reconstructing 3D images in digital holography, using the robust algorithm. A renewed understanding of the nature of 3D reconstruction through intuitive and mathematical analysis is presented. A sparsity based optimization framework is then proposed to solve the 3D reconstruction problem from a single de-focused digital hologram. The proposed methodology is observed to provide 3D phase recovery, of weakly scattering objects like biological cells, with axially

localized volume. The imaging performance as demonstrated in the thesis with optimization based formulation, is in principle not possible to achieve with conventional imaging techniques. We believe that the capability of MGD optimization to handle versatile data sets and imaging configurations in a uniform manner, as demonstrated in the thesis, can make it amenable to various device-based applications in imaging.

सार

आधुनिक इमेजिंग सिस्टम अपरंपरागत हार्डवेयर डिज़ाइन और विशेषज्ञ एल्गोरिदमों का संयोजन हैं। इन सिस्टम में सामान्यतः डेटा एक वस्तु की इमेज से भिन्न होता है, जबकि वास्तव में वह कोडिंग के रूप में होता है। इमेज पुनर्निर्माण (रिकवरी) एल्गोरिदम का कार्य गणितीय समस्या को हल करके इस कोडिंग को उलटना और एक दृश्य व्याख्या योग्य इमेज प्रदान करना है। उलट (इनवर्स) इमेजिंग समस्या को हल करने में मुख्य चुनौती यह होती है कि प्राप्त डेटा अपूर्ण और नोइज़ी होता है। ऐसी समस्याओं को हल करने के लिए एक कुशल फ्रेमवर्क में संयुक्त कॉस्ट फ़ंक्शन को कम करके डेटा-फ़िट त्रुटि और स्पार्सिटी दंड जैसे विशिष्ट उद्देश्यों को ऑप्टिमाइज़ किया जाता है। यह फॉर्मूलेशन अपरंपरागत इमेज पुनर्निर्माण प्राप्त करने की एक बड़ी संभावना प्रदान करता है जो अन्यथा पारंपरिक इमेजिंग तकनीकों के साथ संभव नहीं है। हालाँकि, व्यावहारिक इमेजिंग उपकरणों में ऑप्टिमाइजेशन एल्गोरिदम के अनुप्रयोग को रोकने वाली एक बुनियादी बाधा एक फ्री पैरामीटर की भागीदारी है जिसे रेगुलराईज़ेशन पैरामीटर कहा जाता है। पुनर्प्राप्त इमेज की उत्तमता इस पैरामीटर के चयन पर महत्वपूर्ण रूप से निर्भर करती है। इसके अलावा, पैरामीटर ट्यूनिंग प्रक्रिया में इमेज की उत्तमता का मूल्यांकन करने के लिए व्यक्तिपरक हस्तक्षेप की आवश्यकता होती है। फ्री पैरामीटर को ट्यून करने का चुनौतीपूर्ण कार्य इमेजिंग उपकरणों के उपयोगकर्ताओं को संभवतः उप-इष्टतम प्रदर्शन करने वाली पारंपरिक इमेजिंग तकनीकों पर स्विच करने के लिए प्रेरित कर सकता है। यह थीसिस `मीन ग्रेडिएंट डिसेंट (एमजीडी)' नामक एक नवीनतम ऑप्टिमाइजेशन फ्रेमवर्क प्रस्तावित करके इस समस्या का समाधान करती है, जो इस प्रकार के महत्वपूर्ण फ्री पैरामीटरों को शामिल नहीं करता है। एमजीडी का ध्येय संयुक्त कॉस्ट फ़ंक्शन न्यूनतमकरण प्राप्त करने के बजाय डेटा-फ़िट त्रुटि और रेगुलराइज़ेशन (स्पार्सिटी) उद्देश्यों के बीच संतुलन प्राप्त करने के एक दिलचस्प परिप्रेक्ष्य पर आधारित है। नई पद्धति कम्प्यूटेशनल रूप से लागू करने में सरल है और विभिन्न इनवर्स इमेजिंग समस्याओं की श्रृंखला को हल करने के लिए एक व्यापक फ्रेमवर्क प्रदान करती है। थीसिस में इस प्रस्तावित फ्रेमवर्क को इमेज डिकंवोल्यूशन, डिजिटल होलोग्राफी में क्वांटिटेटिव फेज इमेजिंग और त्रि-आयामी (3डी) इमेजिंग की महत्वपूर्ण इनवर्स इमेजिंग समस्याओं को सफलतापूर्वक हल करने के लिए प्रयोग किया गया है। एमजीडी के रोबस्ट ऑप्टिमाइजेशन फ्रेमवर्क के साथ, एकल-शॉट होलोग्राम डेटा से शॉट- नॉइज़ सीमा से बेहतर फेज यथार्थता और पूर्ण डिटेक्टर रिज़ॉल्यूशन के साथ फेज प्राप्त करना संभव है। यह थीसिस एमजीडी एल्गोरिदम का उपयोग करके डिजिटल होलोग्राफी में 3डी इमेज के पुनर्निर्माण की इमेजिंग समस्या से संबंधित दिलचस्प और मौलिक प्रश्नों पर प्रकाश डालती है। सहज ज्ञान युक्त और गणितीय विश्लेषण के माध्यम से परंपरागत 3डी पुनर्निर्माण की प्रकृति की एक नई समझ प्रस्तुत की गई है। इसके बाद एक स्पार्सिटी आधारित ऑप्टिमाइजेशन फ्रेमवर्क प्रस्तावित किया गया है जो एकल डिफोकस होलोग्राम से 3डी रिकवरी समस्या का समाधान करने के लिए उपयुक्त है। प्रस्तावित पद्धति जैविक कोशिकाओं जैसे प्रकाश का कमजोर प्रकीर्णन (स्कैटरिंग) करने वाले वस्तुओं की 3डी फेज रिकवरी अक्षीय रूप से स्थानीयकृत मात्रा के साथ (विध एक्सिअली लोकलाएजड वॉल्यूम) प्रदान करने में सक्षम है। थीसिस में यह स्पष्ट सिद्ध किया गया है कि जो इमेजिंग परफॉरमेंस प्रस्तावित ऑप्टिमाइजेशन आधारित फॉर्मूलेशन से प्राप्त की जा

सकती है, वह पारंपरिक इमेजिंग तकनीकों के साथ हासिल करना असंभव है। हमारा मानना है कि एमजीडी फ्रेमवर्क की विभिन्न डेटा सेट और इमेजिंग कॉन्फ़िगरेशन के लिये इष्टतम इमेज रिकवरी प्रदान करने की क्षमता, जैसा कि थीसिस में दिखाया गया है, इसे इमेजिंग में विभिन्न डिवाइस-आधारित अनुप्रयोगों के लिए उपयुक्त बना सकती है।

Contents

List of Figures

List of Tables

List of Abbreviations

ADAM Adaptive Moment Estimation

ANN Artificial Neural Network

ASD-POCS Alternating Steepest Descent-Projection onto Convex Set

BPM Beam Propagation method

CCD Charged Coupled Device

CGH Computer Generated Hologram

CMOS Complementary Metal-Oxide Semiconductor

DH Digital Holography

DHM Digital Holographic Microscope

DNN Deep Neural Network

FDT Fourier Diffraction Theorem

FISTA Fast iterative Shrinkage Thresholding Algorithm

FTM Fourier Transform Method

GCV Generalized Cross Validation

KKT Kahrush-Kuhn-Tucker

LS Least Squares

MAP Maximum *a posteriori* Estimator

MGD Mean Gradient Descent

ML	Machine Learning
MLE	Maximum Likelihood Estimator
ODT	Optical Diffraction Tomography
PSF	Point Spread Function
PSM	Phase Shift Method
RBC	Red Blood Cell
RI	Refractive Index
RMSE	Root Mean Square Error
SGD	Stochastic Gradient Descent
SNR	Signal to Noise Ratio
SSIM	Structural Similarity Index Measure
TSV	Total Squared Variation
TV	Total Variation
VI	Variational Inequality
WLS	Weighted Least Squares