

**MEAN GRADIENT DESCENTA
NEW OPTIMIZATION APPROACH WITH
APPLICATIONS TO INVERSE PROBLEMS IN IMAGING**

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**DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
JUNE 2023**

Mean Gradient Descent-
A New Optimization Approach with Applications to Inverse
Problems in Imaging

by

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submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy
to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

JUNE 2023

Dedicated to my teachers and family

CERTIFICATE

This is to certify that the thesis entitled, “**Mean gradient descent- A New Optimization Approach with Applications to Inverse Problems in Imaging**”, being submitted by **Sunaina**, to the Indian Institute of Technology Delhi, for the award of degree of **Doctor of Philosophy** in the Department of Physics is a record of bonafide research work carried out by her under our supervision and guidance. She has fulfilled the requirements for the submission of the thesis, which to the best of our knowledge has reached the required standard.

The material contained in the thesis has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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ACKNOWLEDGEMENTS

I wish to express my heartfelt gratitude to my PhD advisor **Prof. Kedar Khare** for his unwavering support, exceptional guidance and his devoted effort throughout my PhD journey. His critical feedback, deep insights, and encouragement have been invaluable in shaping my ideas and producing a high-quality thesis. His astute and perceptive approach to research was crucial in successfully completing this thesis. I deeply appreciate the valuable lessons he imparted specifically on science ethics and human values. His unwavering positivity, exceptional guidance and kind encouragement has helped me to evolve both as a researcher and as a person. It is a great privilege to be his student.

I would also like to thank my committee members **Prof. P. Senthilkumaran**, **Prof. Alok Sinha**, **Prof. Manideepa Banerjee**, and **Prof. Kedar khare**, for their insightful feedback and constructive criticism. Their comments and suggestions have helped me to refine my work and make it more impactful. I am immensely grateful to **Prof. P. Senthilkumaran** and **Prof. Joby Joseph** for providing me with interesting and engaging lectures in Optics.

I would like to express my profound gratitude to the teachers and professors during my school and graduate studies who played a crucial role in shaping my formative years. I am grateful to my Mathematics teacher in Guru Nank International School, Ludhiana: **Mr. Naveen Bansal** whose interesting lessons have changed the trajectory of my scientific career. I express my deepest gratitude to **Prof. Mohammad Sajjad Athar** who has been a constant source of inspiration all through my life. I am thankful to **Prof. Abbas Ali**, **Prof. Mohammad Shoeb** for their wonderful lectures in my graduate studies from Aligarh Muslim University.

I am thankful to the Indian Institution of Technology Delhi for providing me with a five-year institute assistantship and financial assistance to participate in national and international conferences. Additionally, I appreciate the efforts of the IIT Delhi hostel staff for taking care of the hostel residents in all possible aspects.

I would like to acknowledge my colleagues **Priyanka Lohchab**, **Apoorv Pant**, **Mansi Butola**, **Ritika Malik**, for all the interesting discussions. I am thankful to **Mansi Butola** and **Jasleen Birdi** for being a wonderful collaborators.

I would also like to appreciate my team members of IIT Delhi student chapter of Optica, **Anisha Pathak**, **Shashank Gahlaut**, **Nabarun Saha**, **Sugeet Sunder**, **Sub-**

hajit Karmakar, Kalpak Gupta, Vikas Kumar, Vivek Semwal, Hemant Kumar Singh, Omshankar, Akanksha Angural and Sukhwinder; who have been the part of exciting activities including science outreaches, optics and photonics seminars, IONS 2020 conference organized at IIT Delhi and several others.

I am incredibly grateful to my dear friends, **Mansi Butola, Hemant Kumar Singh, Shilpa Sharma, Milton Mondal, Pragati Sharma, Preeti**, and **Anisha Pathak**, for their invaluable contribution to this journey, which has been both enriching and enjoyable. I consider them all as a real treasure in my life. I am grateful to my family for their love and support. My mother's (**Smt. Meena Kumari**) lovely smiles and father's (**Sh. Gajendra Singh**) faith in me has been a constant source of encouragement throughout my PhD years. It is due to the immense love and blessings of my grandfather, **Sh. Daryao Singh**, that I am where I am today. My siblings, **Vishal Kumar** and **Anjali**, are not only my most significant critics but also my source of motivation and support.

I extend my deepest gratitude to **Haimanthi Mukhopadhyay** for introducing me to the real essence of Yoga in life. Undoubtedly, the practice of yoga has contributed to making my PhD years a smoother experience. I am indebted for the precious teachings and practices which will accompany for my whole life.

Once again, I express my sincere appreciation to everyone who have devoted their precious time and energy for making my PhD years worthwhile.

Sunaina

ABSTRACT

Modern imaging systems are the synergic meld of unconventional hardware design and sophisticated algorithms. The typical data in these systems do not even visually resemble the image of an object whereas it is in a coded form. The task of image reconstruction algorithms is to invert this coding by solving a mathematical problem and provide a visually interpretable image. The major challenge in solving the inverse imaging problem is that the acquired data is incomplete and noisy. An efficient framework to solve such problems involves optimization of specific objectives like data-fit error and sparsity penalty by minimizing a combined cost function. This formulation offers a great possibility to achieve unconventional image reconstruction which is otherwise not possible with traditional imaging techniques. However, a basic roadblock preventing the application of optimization algorithms in practical imaging devices is the involvement of a free parameter called regularization parameter. The quality of the recovered object is critically dependent on its choice. Moreover, the tuning procedure involves subjective intervention for image quality assessment. The daunting task of tuning the free parameter can intimidate the end-users to possibly switch to naive imaging techniques with sub-optimal performance. The thesis addresses this problem by proposing a novel optimization framework of ‘mean gradient descent (MGD)’ which does not involve such critical free parameters. The idea of MGD is based on an interesting perspective of attaining a balance between the data-fit error and regularization terms instead of attaining a total cost minimization. The new methodology is simple to implement computationally and provides a generic framework to solve a range of inverse problems in imaging. The proposed framework is observed to successfully solve the important inverse imaging problems of image deconvolution, quantitative phase imaging and the problem of 3D imaging in digital holography. With the robust and adaptive framework of MGD, it is possible to get phase recovery with full detector resolution and accuracy better than shot-noise limit from a single-shot hologram data. The thesis further delves into intriguing and fundamental inquiries concerning the concept of reconstructing 3D images in digital holography, using the robust algorithm. A renewed understanding of the nature of 3D reconstruction through intuitive and mathematical analysis is presented. A sparsity based optimization framework is then proposed to solve the 3D reconstruction problem from a single de-focused digital hologram. The proposed methodology is observed to provide 3D phase recovery, of weakly scattering objects like biological cells, with axially

localized volume. The imaging performance as demonstrated in the thesis with optimization based formulation, is in principle not possible to achieve with conventional imaging techniques. We believe that the capability of MGD optimization to handle versatile data sets and imaging configurations in a uniform manner, as demonstrated in the thesis, can make it amenable to various device-based applications in imaging.

सार

आधुनिक इमेजिंग सिस्टम अपरंपरागत हार्डवेयर डिज़ाइन और विशेषज्ञ एल्गोरिदमों का संयोजन हैं। इन सिस्टम में सामान्यतः डेटा एक वस्तु की इमेज से भिन्न होता है, जबकि वास्तव में वह कोडिंग के रूप में होता है। इमेज पुनर्निर्माण (रिकवरी) एल्गोरिदम का कार्य गणितीय समस्या को हल करके इस कोडिंग को उलटना और एक दृश्य व्याख्या योग्य इमेज प्रदान करना है। उलट (इनवर्स) इमेजिंग समस्या को हल करने में मुख्य चुनौती यह होती है कि प्राप्त डेटा अपूर्ण और नोइज़ी होता है। ऐसी समस्याओं को हल करने के लिए एक कुशल फ्रेमवर्क में संयुक्त कॉस्ट फ़ंक्शन को कम करके डेटा-फ़िट त्रुटि और स्पार्सिटी दंड जैसे विशिष्ट उद्देश्यों को ऑप्टिमाइज़ किया जाता है। यह फॉर्मूलेशन अपरंपरागत इमेज पुनर्निर्माण प्राप्त करने की एक बड़ी संभावना प्रदान करता है जो अन्यथा पारंपरिक इमेजिंग तकनीकों के साथ संभव नहीं है। हालाँकि, व्यावहारिक इमेजिंग उपकरणों में ऑप्टिमाइज़ेशन एल्गोरिदम के अनुप्रयोग को रोकने वाली एक बुनियादी बाधा एक फ्री पैरामीटर की भागीदारी है जिसे रेगुलराइज़ेशन पैरामीटर कहा जाता है। पुनर्प्राप्त इमेज की उत्तमता इस पैरामीटर के चयन पर महत्वपूर्ण रूप से निर्भर करती है। इसके अलावा, पैरामीटर ट्यूनिंग प्रक्रिया में इमेज की उत्तमता का मूल्यांकन करने के लिए व्यक्तिपरक हस्तक्षेप की आवश्यकता होती है। फ्री पैरामीटर को ट्यून करने का चुनौतीपूर्ण कार्य इमेजिंग उपकरणों के उपयोगकर्ताओं को संभवतः उप-इष्टतम प्रदर्शन करने वाली पारंपरिक इमेजिंग तकनीकों पर स्विच करने के लिए प्रेरित कर सकता है। यह थीसिस 'मीन ग्रेडिएंट डिसेंट (एमजीडी)' नामक एक नवीनतम ऑप्टिमाइज़ेशन फ्रेमवर्क प्रस्तावित करके इस समस्या का समाधान करती है, जो इस प्रकार के महत्वपूर्ण फ्री पैरामीटरों को शामिल नहीं करता है। एमजीडी का ध्येय संयुक्त कॉस्ट फ़ंक्शन न्यूनतमकरण प्राप्त करने के बजाय डेटा-फ़िट त्रुटि और रेगुलराइज़ेशन (स्पार्सिटी) उद्देश्यों के बीच संतुलन प्राप्त करने के एक दिलचस्प परिप्रेक्ष्य पर आधारित है। नई पद्धति कम्प्यूटेशनल रूप से लागू करने में सरल है और विभिन्न इनवर्स इमेजिंग समस्याओं की श्रृंखला को हल करने के लिए एक व्यापक फ्रेमवर्क प्रदान करती है। थीसिस में इस प्रस्तावित फ्रेमवर्क को इमेज डिकंवोल्यूशन, डिजिटल होलोग्राफी में क्वांटिटेटिव फेज इमेजिंग और त्रि-आयामी (3डी) इमेजिंग की महत्वपूर्ण इनवर्स इमेजिंग समस्याओं को सफलतापूर्वक हल करने के लिए प्रयोग किया गया है। एमजीडी के रोबस्ट ऑप्टिमाइज़ेशन फ्रेमवर्क के साथ, एकल-शॉट होलोग्राम डेटा से शॉट- नॉइज़ सीमा से बेहतर फेज यथार्थता और पूर्ण डिटेक्टर रिज़ॉल्यूशन के साथ फेज प्राप्त करना संभव है। यह थीसिस एमजीडी एल्गोरिदम का उपयोग करके डिजिटल होलोग्राफी में 3डी इमेज के पुनर्निर्माण की इमेजिंग समस्या से संबंधित दिलचस्प और मौलिक प्रश्नों पर प्रकाश डालती है। सहज ज्ञान युक्त और गणितीय विश्लेषण के माध्यम से परंपरागत 3डी पुनर्निर्माण की प्रकृति की एक नई समझ प्रस्तुत की गई है। इसके बाद एक स्पार्सिटी आधारित ऑप्टिमाइज़ेशन फ्रेमवर्क प्रस्तावित किया गया है जो एकल डिफोकस होलोग्राम से 3डी रिकवरी समस्या का समाधान करने के लिए उपयुक्त है। प्रस्तावित पद्धति जैविक कोशिकाओं जैसे प्रकाश का कमजोर प्रकीर्णन (स्कैटरिंग) करने वाले वस्तुओं की 3डी फेज रिकवरी अक्षीय रूप से स्थानीयकृत मात्रा के साथ (विध एक्सिअली लोकलाएजड वॉल्यूम) प्रदान करने में सक्षम है। थीसिस में यह स्पष्ट सिद्ध किया गया है कि जो इमेजिंग परफॉरमेंस प्रस्तावित ऑप्टिमाइज़ेशन आधारित फॉर्मूलेशन से प्राप्त की जा

सकती है, वह पारंपरिक इमेजिंग तकनीकों के साथ हासिल करना असंभव है। हमारा मानना है कि एमजीडी फ्रेमवर्क की विभिन्न डेटा सेट और इमेजिंग कॉन्फ़िगरेशन के लिये इष्टतम इमेज रिकवरी प्रदान करने की क्षमता, जैसा कि थीसिस में दिखाया गया है, इसे इमेजिंग में विभिन्न डिवाइस-आधारित अनुप्रयोगों के लिए उपयुक्त बना सकती है।

Contents

List of Figures	xi
List of Tables	xxi
List of Abbreviations	xxiii
1 Introduction	1
1.1 Computational imaging systems	1
1.2 Inverse problem framework	4
1.3 Aim of the thesis	7
1.4 Organization of thesis	9
2 Optimization Methods in Image Reconstruction	11
2.1 Image deblurring problem	11
2.1.1 Image deblurring: an ill-posed inverse problem	13
2.1.2 Inversion methods	15
2.2 Iterative optimization framework	17
2.2.1 Challenge of tuning regularization parameter	20
2.3 Prior developments	23
2.3.1 Automated choice of regularization parameter	24
2.3.2 Modeling the objectives	26
2.3.3 Line search methods	30
2.3.4 Step length	32
2.3.5 Stochastic gradient descent	34
2.3.6 Adam	35
2.3.7 Automatic differentiation	37
2.3.8 Nesterov accelerated gradient descent	38

2.3.9	Fast iterative shrinkage thresholding algorithm (FISTA)	39
2.4	Discussion and conclusions	40
3	New Optimization Framework of Mean Gradient Descent	43
3.1	A new perspective on the optimization formulation	43
3.2	Mean gradient descent (MGD) methodology	48
3.3	Simulation results for image deconvolution	50
3.3.1	Step size selection in MGD iterations	51
3.4	Automated behavior of MGD	55
3.5	Experimental results	59
3.6	MGD: a generic framework	67
3.7	Conclusions	72
4	Single-shot Interferogram Analysis	75
4.1	Historical overview of holography	76
4.2	Traditional phase reconstruction with FTM	78
4.3	Interferogram analysis as an inverse problem	84
4.4	Demodulation of off-axis hologram by MGD optimization: simulation results	88
4.5	Demodulation of on/ near on-axis hologram: simulation results	95
4.6	Conclusions	97
5	True 3D Reconstruction in Digital Holography	99
5.1	Introduction	99
5.2	Nature of 3D reconstruction in holography	103
5.2.1	Physical intuition	103
5.2.2	Mathematical analysis	104
5.3	True 3D reconstruction using inverse problem framework	107
5.3.1	Simulations and results	109
5.4	Discussion	116
5.5	Conclusions	117
6	3D Reconstruction of Unstained Weakly Scattering Cells	119
6.1	Introduction	119
6.2	Prior literature overview	120
6.3	Problem overview	125

6.3.1	Experimental set-up and problem geometry	125
6.3.2	Forward imaging model	129
6.3.3	Numerical computation of de-focus distance	129
6.3.4	Peculiar nature of Back-propagated object field	131
6.4	Numerical reconstruction of 3D complex field of normal RBCs	132
6.4.1	Amplitude contrast: a new weight function in iterative reconstruction	136
6.4.2	3D recovery of malaria-infected red blood cells	137
6.4.3	Axial localization in polystyrene micro-sphere beads	141
6.5	Discussion and future avenues	141
6.6	Conclusions	142
7	Conclusions and Future Directions	145
7.1	Conclusions	145
7.2	Future Directions	147
A	Mathematical Preliminaries	149
A.1	Convex sets and functions	149
A.2	Delta function	150
A.3	Linear and space invariant imaging system	151
A.4	Euler Lagrange formalism	152
A.4.1	Computation of functional gradients	153
A.4.2	Wirtinger derivatives	153
A.5	Bayesian framework	154
A.6	Wiener filter	156
A.7	Thin phase grating condition in Tomography	157
B	Pseudocodes	159
C	Supplementary details	163
C.1	Variational inequality formulation of optimization problem	163
C.2	Demodulation of Off-axis hologram	164
C.3	Metrics to estimate defocus distance	164
	References	170

List of Figures

1.1	Computational imaging system. The data has no similarity with the image of the object of interest. The inverse problem is solved computationally to get a visually interpretable image.	3
1.2	The modern computational imaging systems that rely on the inverse problem framework. The typical raw data in these systems is displayed in the respective insets.	3
2.1	(a) Ground-truth image of cameraman defined over the computational window of size 256×256 pixels. (b) 2D Gaussian function of FWHM 4×4 pixels taken as PSF. Blurred noisy images simulated using Poisson random noise corresponding to the average light level of, (c) 10^4 photons/pixel and, (d) 10^3 photons/pixel.	14
2.2	(a) Inverse filter solution. (b) Image obtained on convolution of PSF in Fig. 2.1 (b) with the solution in (a). (c) Weiner filter solution. Solutions obtained after, (d) 10, (e) 30 and, (f) 50 data error reduction iterations as described in Eq. (2.8)	16
2.3	Image deconvolution by the traditional α parameter tuning based optimization approach. deblurred solutions for the α values of (a) 10^{-2} , (b) 0.5×10^{-3} , and (c) 10^{-4} respectively corresponding to the blurred image in Fig. 2.1(c) simulated with mean photon counts per pixel of 10^4 . Solutions obtained for the blurred image simulated with mean photon counts per pixel of 10^3 as in Fig. 2.1(d) when α is chosen to be (d) 0.5×10^{-2} (optimal α changed for this case), (e) 0.5×10^{-3} , and (f) 10^{-3}	22
2.4	(a) Relative solution error E_{sol} and (b) SSIM index, plotted against different $\log_{10}\alpha$ values for both the cases of simulated image data shown in Figs. 2.1 (c) and (d).	23

3.1	Optimization as a force balancing problem.	47
3.2	Schematics of MGD progression	50
3.3	(a) Blurred noisy image. (b) Deconvolved image after 600 MGD iterations. (c) The plot of angle θ versus iteration number. (d) Progression of relative error between the solution iterate and the ground truth image	51
3.4	Progression of, (a) data fidelity (C_1), and (b) total variation C_2 with MGD iterations. The plots are shown on the log scale.	52
3.5	Stagnation of function due to large step size	54
3.6	Image deblurring by MGD algorithm. The deblurred solutions from the blurred noisy data shown corresponding to mean photon counts of (a) 10^4 photons/pixel, data in Fig. 2.1(c), and (b) 10^3 photons/pixel, data in Fig. 2.1(d), after 400 MGD iterations. Intensity profiles plotted along the red line (c) in image (a) and, (d) in the image (b). The plots along the cor- responding pixels in the ground-truth and blurred images are shown with black and blue curves respectively. (e).	57
3.7	(a) Angle between \hat{u}_1 and \hat{u}_2 , logarithm of (b) data fidelity term, and (c) TV term plots against the iteration number for both the noise levels. . . .	58
3.8	Images of size 310×310 pixels recorded at the focused position from a bright-field microscope for (a) higher and (b) lower intensity of the illumi- nation, denoted by illumination 1 and illumination 2 respectively. (c) The point spread function of the microscope system obtained from PSFgenerator plugin of ImageJ. (d) The plot showing the difference in the fluctuations along the dotted lines in the middle flat regions of recorded images (a) and (b). The images in (a) and (b) are shown with the window of (mean $\pm 3 \times$ standard deviation).	61
3.9	Illustration of MGD algorithm applied to experimental data of the case illumination 1, recorded using a bright-field microscope. (a) The deblurred recovery after 400 MGD iterations. (b) Zoomed regions on (a) marked with red rectangle, (c) corresponding regions of the recorded images that are blurred due to system PSF. (d) The profiles along red and blue lines in (b), (c). (e) Profiles along dotted red lines in (a) and corresponding pixels in the recorded blurred images. Image (a) is shown with the color bar of window level mean $\pm 3 \times$ standard deviation.	62

3.10	Illustration of MGD algorithm applied to experimental data of the case illumination 2, recorded using a bright-field microscope. (a) The deblurred recovery after 400 MGD iterations. (b) Zoomed regions on (a) marked with red rectangle, (c) corresponding regions of the recorded images that are blurred due to system PSF. (d) The profiles along red and blue lines in (b), (c). (e) Profiles along dotted red lines in (a) and corresponding pixels in the recorded blurred images. Image (a) is shown with the color bar of window level $\text{mean} \pm 3 \times \text{standard deviation}$	63
3.11	For illumination 1, the Fourier magnitudes of (a) recorded image, and (b) deblurred image. For illumination 2, the Fourier magnitudes of (c) recorded image, and (d) deblurred image. For the display purpose Fourier magnitudes ($ F $) are shown as $ F ^{0.25}$. The dotted circles in the figures show location of the cut-off frequency of the OTF as estimated from the NA of the microscope objective for $\lambda = 0.5 \mu\text{m}$	64
3.12	(a) The recorded cervical cell image of size 350×350 . The inset shows the PSF of dimension 11×11 generated by PSFgenerator tool. The deblurred images after (c) 400 MGD iterations and (d) 10 RL iterations. (b) The profiles along the dotted lines in (a), (c), and (d). The inset is the zoom-in version of the edge and shows that MGD deblurred image has sharper edge as compared to that of RL. In RL solution (d) along with the grainy artifacts, the ghosting at the edges can also be seen, for example, as indicated by the arrow and the insets in (c), (d) that show the zoom-in version of the corresponding features.	66
3.13	(a) Blurred cameraman image with additive Gaussian noise, and (b) deblurred image with MGD optimization	68
3.14	(a) Ground-truth, (b) blurred image with additive Gaussian noise, and (c) deblurred image with MGD optimization; for the test object of moon. (d) Ground-truth, (e) blurred image with additive Gaussian noise, and (f) deblurred image with MGD optimization; for the test object of penguin.	69
3.15	(a) Blurred cameraman image simulated with Poisson noise corresponding to the average light level of 10^2 photons/ pixel. Recovered images with, (b) least squares data fidelity, and (c) negative of log-likelihood function used as data fidelity.	70

3.16	Effect of choice of regularization functionals. (a) Blurred Cameraman image simulated with Poisson random noise with an average light level of 10^4 photons/ pixel. Image recoveries after 400 MGD iterations, when least-squares is used as data fidelity and the regularization term, (b) TV_1 , (c) TV_2 , and (d) TV_3 ; are used in the MGD algorithm.	71
4.1	Off-axis holography. (a) Recording of hologram on a photographic film in off-axis reference wave configuration. (b) Reconstruction of object by illumination the hologram with conjugate reference wave.	78
4.2	Digital holography	79
4.3	Fourier transform method	80
4.4	Sampling and numerical aperture of hologram	81
4.5	(a) Phase map of square-shaped object with a step phase of $2\pi/3$ radians over 250×250 pixels defined on a 500×500 pixel grid. (b) Hologram of the object in (a) with a tilted plane reference beam simulated with Poisson noise corresponding to average light level of 10^4 photons/pixel. (c) Zoomed-in version of Fourier magnitude of hologram showing the circular filter of radius 0.5 times the distance between dc and cross term peaks. For display purpose the square root of Fourier magnitude of the hologram is shown, (d) Phase image reconstructed using the Fourier filtering method. Inset in (c) shows a zoom-in version of its central part indicating the spatial frequency coordinates, cross-terms, and the circular filter.	83
4.6	Recovered phase maps corresponding to the α value of, (a) 0.01, (b) 0.5, and (c) 10. (d)-(f) Profile plots along the blue dotted lines in (a)-(c) respectively.	88
4.7	(a) Phase and (b) amplitude of solution for object wave after 200 MGD iterations with step size t kept fixed.	90
4.8	Behaviour of logarithm of (a) relative hologram error $C_{err}/\ H\ _2^2$, and (b) C_{TV} , with iteration number; for three light levels of 10^3 , 10^4 and 10^5 photons/pixel. The magenta line shows the ground-truth TV for the ideal step phase object shown in Fig. 1(a). (c) Variation of angle(θ) with iteration number corresponding to three light levels of 10^3 , 10^4 and 10^5 photons/pixel.	91

4.9	Progression of solution with MGD algorithm for Poisson noise realization with light level of 10^4 photons/pixel. Amplitude of the solution after (a) 500 and (b) 2000 iterations. Phase reconstruction after (c) 500 and (d) 2000 iterations. (e) Phase profile of the resultant solution in (d) along the dotted line. Note that the solution contains sharp edges as compared to FTM solution shown in Fig. 1(d).	92
4.10	Amplitude, phase, and the phase profile of the resultant solution after 2000 MGD iterations for (a)-(c) 10^3 photons/pixels, and (d)-(f) 10^5 photons/pixels light level case. The phase profiles are plotted along the shown dotted blue lines.	93
4.11	Hologram of step phase object with (a) on-axis and (b) near on-axis spherical reference wave simulated with Poisson noise with an average light level of 10^4 photons/pixel. (c) , (d) Fourier magnitudes of the holograms in (a) and (b) respectively showing overlap between dc and cross terms. (e) , (g) and (f) , (h) are the reconstructed phase maps and their corresponding phase profiles along the dotted lines respectively after 2500 MGD iterations.	96
5.1	Pictorial representation of hologram recording and replay processes. (a) Hologram recording of a 3D object, consisting of 4 reflective point objects located at different planes, with a tilted plane reference wave. (b) Back-propagation of the known 2D object field at the hologram plane to the different planes of the 3D volume. In each plane, one can clearly see the focused as well as the de-focused field distribution. In (b), colors are used for the purpose of representing the de-focusing effect.	100
5.2	Pictorial representation of hologram recording and backpropagation of the object field to the original object volume. The operator \hat{A} denotes the hologram formation process which takes information about a 3D volume to a 2D detector plane. The back-propagation of the object field is seen to be equivalent to the adjoint or \hat{A}^\dagger operation.	106

5.3	Schematic flow-chart of methodology for 3D image reconstruction from recorded hologram. The traditional method uses back-propagation of object field $V(x, y)$ in hologram plane for 3D image estimation. The proposed approach uses a regularized optimization method for performing a superior 3D image reconstruction.	107
5.4	Problem geometry showing the 3D object \tilde{U} to be estimated and the data $V(x, y)$ at the detector plane.	110
5.5	(a), (d) Magnitudes of amplitude and phase objects shown as 3D rendering, (b), (c) real and imaginary parts of the object field $V(x, y)$ at the detector plane due to the amplitude object, (e), (f) real and imaginary parts of the object field $V(x, y)$ at the detector plane due to the phase object, (g), (h) real and imaginary parts of the phase object for four specific planes where the letters A, B, C, D are located.	112
5.6	3D reconstruction of the amplitude object with four small reflectors. (a), (b) 3D rendering of the amplitude of the recovered point object obtained by simple backprojection of the object field and by using the iterative reconstruction respectively. (c), (d) Image of the planes where the point objects originally existed in the 3D object box shown in (a) and (b) respectively. . .	114
5.7	3D reconstruction of the text phase object. (a), (b) 3D rendering of amplitude of the reconstruction using simple back-projection and using proposed iterative reconstruction. (c), (d) Real and imaginary parts for four different planes where the letters were located in the true 3D object box are shown for the reconstructions in (a), (b) respectively.	115
5.8	(a), (b) Relative reconstruction error for the two illustrations shown in Fig. 5.6 and Fig. 5.7 as a function of iteration number respectively.	116
6.1	Fourier diffraction theorem	122
6.2	System schematic of reconstruction problem. The planes shown with red solid lines correspond to the location of the RBC and its defocused image. Blue dotted lines refer to the perfect focus plane and its conjugate at the detector. Focal lengths f_1 and f_2 are not shown to scale.	125
6.3	Digital Holographic Microscope (DHM)	126

6.4	Problem geometry for the three illustrated cases of healthy RBCs, malaria-infected RBCs, and polystyrene microsphere beads.	128
6.5	(a) ROI of the defocused RBC hologram recorded at 40x magnification in digital holographic microscope. (b)-(c) Unwrapped phase and amplitude of the 2D complex field $V(x, y)$ reconstructed with FTM. Amplitude is shown after background subtraction and Gaussian windowing. (d) The plot of amplitude contrast metric $\sqrt{\sigma}$ v/s the z-distance from the detector plane. The blue arrow in (d) shows that the amplitude contrast is minimum and hence defocus distance is at $9\mu m$	130
6.6	(a) The amplitude of back-propagated 2D complex field $V(x, y)$ at various z- distances in the range ($0 - 18\mu m$) shown in the x-z plane, which cuts through the center of y axis. Note that in (a) $0\mu m$ represents the detector plane. The corresponding amplitude maps in x-y planes located at, (b) $12\mu m$, (c) $9\mu m$ and, (d) $6\mu m$ z-distances from the detector plane as marked by the three dotted magenta lines in (a).	132
6.7	Amplitude maps in each of the 5 planes of the reconstructed RBC volume corresponding to the, (a)-(e) back-propagated 3D complex field (U_B), (f)-(j) optimization reconstruction U_O , and (k) -(o) the solution U_{OW} obtained with inverse amplitude contrast ($1/\sqrt{\sigma}$) as a weight metric in MGD iterations. Note that each row is represented by a common scale bar.	134
6.8	Unwrapped phase maps in each of the 5 planes of the reconstructed RBC volume corresponding to, (a)-(e) back-propagated 3D complex field U_B , (f)-(j) optimization reconstruction U_O , and (k) -(o) the solution U_{OW} obtained with inverse amplitude contrast ($1/\sqrt{\sigma}$) as a weight function in MGD iterations. A common scale bar is used to display all the phase maps. . . .	135
6.9	Sideways view in x-z and y-z planes of the amplitudes corresponding to the complex fields, (a) U_B , (b) U_O and, (c) U_{OW} . Scale bars for each of the amplitude maps are same as shown in Fig. 6.7. Sideways view in x-z and y-z plane of the unwrapped phase maps corresponding to the complex fields (d) U_B , (e) U_O , and (f) U_{OW} . Scale bars for each of the phase maps are same as shown in Fig. 6.8.	137

6.10	(a) Bright-field image of the normal RBC and the RBC infected with malaria parasite as indicated by red arrow, both imaged in the same ROI. (b) The corresponding hologram recorded at 40x magnification on DHM. (c) Amplitude, and (d) phase maps of the 2D complex field $V(x, y)$ at the detector plane.	138
6.11	(a)-(e) Amplitude, and (f)-(j) phase maps of the back-propagated field U_B across 1-5 planes of RBC volume. (k)-(o) Amplitude, and (p)-(t) phase maps of the complex field U_{OW} , obtained by weights assisted optimization, across 1-5 planes of reconstructed RBC volume. The red arrow in (q) shows the phase dip at the location of parasite.	138
6.12	(a) ROI of the defocused hologram for $10\mu m$ polystyrene micro-sphere recorded at 40x magnification in digital holographic microscope. (b) The plot of amplitude contrast metric $\sqrt{\sigma}$ v/s the z-distance from the detector plane. The blue arrow in (b) shows that the amplitude contrast is minimum and hence defocus distance is at $7.5\mu m$. (c), (d) The unwrapped phase and amplitude, respectively of the 2D complex field $V(x, y)$ reconstructed with FTM. Amplitude is shown after background subtraction and Gaussian windowing. Sideways view in x-z plane of the amplitudes corresponding to the complex fields, (e) U_B and, (f) U_{OW} . Sideways view in x-z plane of the unwrapped phase corresponding to the complex fields, (g) U_B and, (h) U_{OW} .	140
A.1	(a) Convex set. (b) Non-convex set. (c) 1D Convex function.	150
A.2	Thin phase grating	157
C.1	Reconstructed amplitude, phase, and the profile plot along the dotted line, (a)-(c) after 250 MGD iterations, and (d)-(f) after 600 MGD iterations; for the off-axis hologram simulated with Poisson random noise with an average light level of 10^4 photons/pixel	165
C.2	The graph of (a) angle (θ) between the gradient descent directions $-\hat{\mathbf{u}}_1$ and $-\hat{\mathbf{u}}_2$, (b) logarithm of relative data domain error, and (c) logarithm of TV of the solution; each plotted against the iteration number. The red, blue, and green colors represent the associated characteristics shown for the hologram simulated with the average light levels of 10^5 , 10^4 , and 10^3 photons/ pixel respectively	166

C.3	Reconstructed amplitude, phase, and the profile plot along the dotted line for the cases of off-axis hologram simulated with an average light level of, (a)-(c) 10^3 photons/pixel, and (d)-(f) 10^5 photons/pixel	167
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List of Tables

- 3.1 SSIM and relative L_2 -norm errors of the recovered images with respect to ground-truth for noise level corresponding to the average light level of 10^4 photons/pixel and 10^3 photons/pixel. Comparison is drawn for MGD recovered images and the best recovery obtained by adjusting the α value in the traditional optimization. 59
- 4.1 Phase rms error values after 2000 iterations of MGD algorithm corresponding to three cases of noise levels added to the hologram data. The average value of rms phase error for each case is calculated from 10 different realizations of random Poisson noise. . 94

List of Abbreviations

ADAM Adaptive Moment Estimation

ANN Artificial Neural Network

ASD-POCS Alternating Steepest Descent-Projection onto Convex Set

BPM Beam Propagation method

CCD Charged Coupled Device

CGH Computer Generated Hologram

CMOS Complementary Metal-Oxide Semiconductor

DH Digital Holography

DHM Digital Holographic Microscope

DNN Deep Neural Network

FDT Fourier Diffraction Theorem

FISTA Fast iterative Shrinkage Thresholding Algorithm

FTM Fourier Transform Method

GCV Generalized Cross Validation

KKT Karush-Kuhn-Tucker

LS Least Squares

MAP Maximum *a posteriori* Estimator

MGD Mean Gradient Descent

ML Machine Learning

MLE Maximum Likelihood Estimator

ODT Optical Diffraction Tomography

PSF Point Spread Function

PSM Phase Shift Method

RBC Red Blood Cell

RI Refractive Index

RMSE Root Mean Square Error

SGD Stochastic Gradient Descent

SNR Signal to Noise Ratio

SSIM Structural Similarity Index Measure

TSV Total Squared Variation

TV Total Variation

VI Variational Inequality

WLS Weighted Least Squares