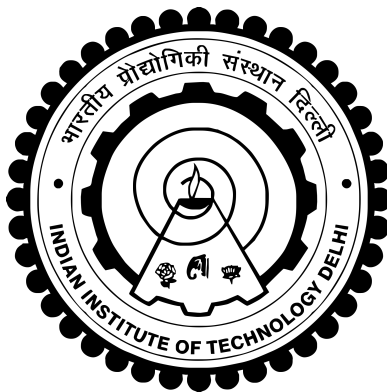


**DESIGN OF GUIDANCE LAWS BASED ON  
TIME-ENERGY EFFICIENT ROBUST CONTROL  
STRATEGIES**

**ARUNAVA BANERJEE**



**DEPARTMENT OF ELECTRICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI  
NOVEMBER 2021**

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**DESIGN OF GUIDANCE LAWS BASED ON  
TIME-ENERGY EFFICIENT ROBUST CONTROL  
STRATEGIES**

*by*

**ARUNAVA BANERJEE**

**Department of Electrical Engineering**

**submitted**

**in fulfilment of the requirements of the degree of Doctor of Philosophy**

**to the**



**INDIAN INSTITUTE OF TECHNOLOGY DELHI**

**NOVEMBER 2021**

To my Maa, Smt. Mina Banerjee,  
and my Baba, Sri. Amitava Banerjee

# Certificate

This is to certify that the thesis entitled “**Design of Guidance Laws Based on Time-Energy Efficient Robust Control Strategies**”, submitted by **Arunava Banerjee** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the original, bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

**Prof. I N Kar** (Caretaker Supervisor)

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# Abstract

The use of non-linear control theory to design guidance laws with the primary objective of intercepting targets has been explored extensively for the last several decades. The solution of this practical problem poses significant challenges due to variation in the system parameters, external disturbances, time-varying uncertainties, limited time and fuel availability etc. The initial high heading error during the launch of an interceptor while dealing with maneuvering targets also makes capture a demanding task for a missile. Thus, consolidated guidance approaches have been proposed in this thesis to achieve successful interception with efficient time-energy utilization and robustness against external disturbances. Modern optimal control and optimization methodologies have been explored to obtain near-optimal guidance laws and to generate reference trajectories which are time-energy efficient. Robust guidance laws are then proposed, which are capable of tackling external disturbances and track these reference trajectories to introduce a sense of time-energy efficiency.

To further improve the overall time-energy utilization of the system, a deployment approach which chooses either a robust guidance law or a near-optimal guidance scheme based on the occurrence of external disturbances is also proposed in the thesis. Additionally, due to constraint on space, weight, and cost, the state-of-art processors whose computational capabilities have increased many folds over the last few decades cannot be generally installed onboard a missile. An efficient way of minimizing the computational burden can be ensured by reducing the updates in the control input, thereby minimizing the load on the onboard processors. Hence, along

with ensuring time-energy minimization and robustness, update reduction is also addressed in this thesis by introducing the concept of event-trigger control and quantized control.

This thesis also incorporates the practical aspect of input saturation for the interceptor which enhances its applicability for practical real-time systems. Simulation studies with non-maneuvering as well as maneuvering targets performing bank-to-bank and step maneuvers in both tail-chase and head-on engagements are included to highlight the efficacy of the guidance strategies. Stability analysis employing Lyapunov method is also carried out to establish the convergence of the system states on application of the proposed control approaches.

**Keywords:** Missile Guidance, Pseudospectral Optimal Control, Nature-Inspired Optimization, Time Delay Control, Time Delayed Estimation, Adaptive-Robust Control, Event-Trigger Control, Quantized Control, Input Saturation.

## सार

लक्ष्य को बाधित करने के प्राथमिक उद्देश्य के साथ मार्गदर्शन कानूनों को डिजाइन करने के लिए गैर-रेखीय नियंत्रण सिद्धांत का उपयोग पिछले कई दशकों से बड़े पैमाने पर किया गया है। इस व्यावहारिक समस्या का समाधान प्रणाली के मापदंडों में भिन्नता, बाहरी डिस्टर्बेंस, समय-भिन्न अनिश्चितताओं, सीमित समय और ईंधन की उपलब्धता आदि के कारण महत्वपूर्ण चुनौतियों का सामना करता है। मनुवेरिंग के लक्ष्यों से निपटने के दौरान एक इंटरसेप्टर के लॉन्च के दौरान प्रारंभिक उच्च शीर्षक त्रुटि भी बनाता है एक मिसाइल के लिए एक मांगलिक कार्य पर कब्जा। इस प्रकार, इस थीसिस में कुशल समय-ऊर्जा उपयोग और बाहरी गड़बड़ी के खिलाफ मजबूती के साथ सफल अवरोधन प्राप्त करने के लिए समेकित मार्गदर्शन दृष्टिकोण प्रस्तावित किए गए हैं। निकट-इष्टतम मार्गदर्शन कानून प्राप्त करने और समय-ऊर्जा कुशल संदर्भ प्रक्षेपवक्र उत्पन्न करने के लिए आधुनिक इष्टतम नियंत्रण और अनुकूलन पद्धतियों का पता लगाया गया है। तब मजबूत मार्गदर्शन कानून प्रस्तावित किए जाते हैं, जो बाहरी गड़बड़ी से निपटने में सक्षम होते हैं और समय-ऊर्जा दक्षता की भावना को पेश करने के लिए इन संदर्भ प्रक्षेपवक्रों को ट्रैक करते हैं।

प्रणाली के समग्र समय-ऊर्जा उपयोग को और बेहतर बनाने के लिए, एक परिनियोजन दृष्टिकोण जो या तो एक मजबूत मार्गदर्शन कानून या बाहरी डिस्टर्बेंस की घटना के आधार पर एक इष्टतम मार्गदर्शन योजना का चयन करता है, थीसिस में भी प्रस्तावित है। इसके अतिरिक्त, जगह, वजन और लागत की कमी के कारण, अत्याधुनिक प्रोसेसर जिनकी कम्प्यूटेशनल क्षमताओं में पिछले कुछ दशकों में कई गुना वृद्धि हुई है, आमतौर पर मिसाइल पर स्थापित नहीं किया जा सकता है। नियंत्रण इनपुट में अद्यतनों को कम करके कम्प्यूटेशनल बोझ को कम करने का एक कुशल तरीका सुनिश्चित किया जा सकता है, जिससे ऑनबोर्ड प्रोसेसर पर लोड को कम किया जा सके। इसलिए, समय-ऊर्जा न्यूनीकरण और मजबूती सुनिश्चित करने के साथ, इस थीसिस में इवेंट-ट्रिगर नियंत्रण और क्वान्टिजेशन नियंत्रण की अवधारणा को पेश करके अद्यतन कमी को भी संबोधित किया गया है।

इस थीसिस में इंटरसेप्टर के लिए इनपुट संतृप्ति का व्यावहारिक पहलू भी शामिल है जो व्यावहारिक रीयल-टाइम सिस्टम के लिए इसकी प्रयोज्यता को बढ़ाता है। मार्गदर्शन रणनीतियों की प्रभावकारिता को उजागर करने के लिए नॉन-मनुवेरिंग के साथ-साथ मनुवेरिंग, और बैंक-टू-बैंक और स्टेप मैनुवर दोनों में टेल-चेज़ और हेड-ऑन एंगेजमेंट के साथ सिमुलेशन अध्ययन शामिल किए गए हैं। प्रस्तावित नियंत्रण दृष्टिकोणों के आवेदन पर सिस्टम राज्यों के अभिसरण को स्थापित करने के लिए लाइपुनोव पद्धति को नियोजित करने वाला स्थिरता विश्लेषण भी किया जाता है।

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# List of Publications

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# Abbreviations

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Abbreviation/Symbol	Description
PN	Proportional Navigation
PPN	Pure Proportional Navigation
TPN	True Proportional Navigation
GPN	Generalized Proportional Navigation
EA	Evolutionary Algorithm
HJ	Hamilton-Jacobi
SMC	Sliding Mode Control
TDC	Time-Delayed Control
TDE	Time-Delayed Estimation
ARC	Adaptive Robust Control
ARTDC	Adaptive Robust Time-Delayed Control
PID	Proportional Integral Derivative
SCA	Sine-Cosine Algorithm
BB-BC	Big-Bang Big-Crunch
DE	Differential Evolution
PSM	Pseudospectral Method
ATERC	Automatic Time-Energy Efficient Robust Control
ET-TDC	Event-Trigger based Time-Delayed Control
QTDC	Quantized Time-Delay Control
PMP	Pontryagins Minimum Principle
LGL	Legendre-Gauss-Lobatto
LGR	Legendre-Gauss-Radau
LG	Legendre-Gauss
GLQ	Gauss-Lobatto Quadrature
DEPN	Differential Evolution based All-Aspect Proportional Navigation
AAPN	All-Aspect Proportional Navigation
BBBCPN	Big-Bang Big-Crunch based All-Aspect Proportional Navigation
COM	Center of Mass
SCAPN	Sine-Cosine Algorithm based All-Aspect Proportional Navigation
PSOGL	Particle Swarm Optimization based Guidance Law
UUB	Uniformly Ultimately Bounded
B-t-B	Bank-to-Bank
NSSMC	Non-Singular Terminal Sliding Mode Control
ETSMC	Event-Triggered Sliding Mode Control

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# Nomenclature

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For Two Dimensional Engagement

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Symbol	Symbol Description
$V_M$	Missile Velocity
$V_T$	Target Velocity
$\alpha_M$	Flight path angle of the missile
$\alpha_T$	Flight path angle of the target
$a_M$	Missile Latax
$a_T$	Target Latax
$R_M = (R_{M_1}, R_{M_2})$	Missile Position
$R_T = (R_{T_1}, R_{T_2})$	Target Position
$\lambda$	Line of sight angle (LOS)
$L$	Lead angle
$h$	Heading angle
$\text{int}(\cdot)$	Greatest integer function
$M_n \in [0 \ 1]$	Randomly generated scalar
$d$	External Disturbances
$\bar{a}_M$	Bound on Control Input
$J$	Cost Function
$t_f$	Time of Flight
$W_t$	Time-Weight

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For Three Dimensional Engagement

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Symbol	Symbol Description
$x_t$	Target Position along X
$y_t$	Target Position along Y
$z_t$	Target Position along Z
$\phi_t$	Yaw angle of Target
$\gamma_t$	Pitch angle of Target
$V_t$	Velocity of Target
$x_m$	Missile Position along X
$y_m$	Missile Position along Y
$z_m$	Missile Position along Z
$\phi_m$	Yaw angle of Missile
$\gamma_m$	Pitch angle of Missile
$V_m$	Velocity of Missile
$g$	Acceleration due to Gravity
$m$	Mass of the missile
$a_{yt}$	Yaw Latax of Target
$a_{pt}$	Pitch Latax of Target
$a_{ym}$	Yaw Latax of Missile
$a_{pm}$	Pitch Latax of Missile
$\phi_l$	Yaw angle of LOS
$\gamma_l$	Pitch angle of LOS
$L_y$	Yaw Lead angle
$L_p$	Pitch Lead angle
$h_y$	Yaw Heading angle
$h_p$	Pitch Heading angle
$\Gamma$	Thrust Force of Missile
$D$	Drag Force of Missile
$\mathbf{d}$	Disturbance vector
$D_0$	Zero Lift Drag
$s$	Plan-form Reference Area
$\rho$	Atmospheric Density
$K$	induced drag co-efficient
$T(K)$	Temperature
$\Gamma$	Thrust
$V_{ty}^L, V_{tz}^L$	Components of Target Velocity along the LOS Frame

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