

**VIBRATION ANALYSIS OF MULTILAYERED BEAMS WITH  
CONSTRAINED VISCOELASTIC LAYERS**

**By  
Nanik. Thakurdas. Asnani  
Department of Mechanical Engineering  
Indian Institute of Technology, Delhi**

**Thesis submitted to the Indian Institute of Technology,  
Delhi, for the award of the degree of Doctor of  
Philosophy in Mechanical Engineering**

**1971**

### ABSTRACT

Equations of motion for flexural vibrations of unsymmetrical beam arrangements of general type with  $n$  constrained viscoelastic layers and  $(n+1)$  constraining elastic layers have been derived using variational method. The detailed analysis of vibration damping characteristics of multilayered beams with simple supports and subjected to harmonic disturbances has been carried out. For this analysis, three criteria viz. constant weight, constant size and constant static stiffness have been introduced. Evaluation of damping effectiveness on the basis of system loss factor and displacement response effectiveness along with weight, size and static stiffness characteristics for the multilayered beams has been done. An approximate method for determination of response of multilayered beams with and without concentrated mass for arbitrary and conditions and subjected to deterministic harmonic disturbances has been evolved. Vibration response tests have been carried out on sandwich beams with fixed-fixed and cantilever type of end conditions, with and without concentrated mass. The theoretical results obtained from the approximate method have been compared with experimental results and thus the validity of the method

has been verified. Experimental work has also been done to determine the dynamic properties in shear of visco-elastic material, which has been used in making the sandwich beam test specimens.

## C E R T I F I C A T E

This is to certify that the thesis entitled "Vibration Analysis of Multilayered Beams with constrained Viscoelastic Layers" being submitted by Mr. Nanik. Thakurdas. Asnani. to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mechanical Engineering, is record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis, which has reached the requisite standard.

The results contained in this thesis have not been submitted in part or in full, to any other university or institute for the award of any degree or diploma.

*B. C. Nakra*

25.8.71

(B. C. Nakra)  
Professor, Mech. Engineering Deptt.  
Indian Institute of Technology,  
Hauz Khas, New Delhi-29.

## A C K N O W L E D G E M E N T S

Foremost thanks are due to Professor B.C. Nakra, who has suggested and supervised this work. His readiness to discuss at all times and his friendly encouragement, have been most valuable. Thanks are also due to Professor B.K. Ahuja, Head of Mechanical Engineering Department and to Professor R.N. Dogra, Director, for providing facilities for this work.

Thanks are due to Mr. R.P. Mahna and Mr. B.B. Arya of Vibration Laboratory for their help during the experimental work. Thanks are also due to staff of Computer Centre for cooperation in computation work. The author wishes to express his appreciation to Mr. D.R. Joshi and Mr. G.S. Bajwa for assistance in preparing this manuscript.

Finally, the author wishes to offer heartfull gratitude and apologies to those near and dear ones for patiently enduring unavoidable difficulties from his preoccupation with this work.

(Nanik. Thakurdas. Asnani)

# C O N T E N T S

| Chapter |                                                                               | Page No. |
|---------|-------------------------------------------------------------------------------|----------|
|         | TERMINOLOGY                                                                   | 1        |
| I       | INTRODUCTION AND REVIEW OF PREVIOUS WORK                                      | 4        |
|         | I.A Introduction                                                              | 4        |
|         | I.B Literature Survey                                                         | 7        |
|         | I.C Outline of present work                                                   | 13       |
| II      | THEORY OF FLEXURAL VIBRATIONS OF MULTILAYERED UNSYMMETRICAL BEAMS             | 18       |
|         | II.1 Multilayered Beam Arrangement                                            | 18       |
|         | II.2 Assumptions                                                              | 18       |
|         | II.3 Derivation of the Differential Equations of Motion                       | 19       |
|         | II.4 Boundary Conditions                                                      | 33       |
| III     | RESPONSE AND DAMPING OF UNSYMMETRICAL MULTILAYERED BEAMS WITH SIMPLE SUPPORTS | 36       |
|         | III.1 Solution for Simply Supported Beam with Sinusoidal Excitation           | 36       |
|         | III.2 Damping Effectiveness                                                   | 39       |
|         | III.3 General Multilayered Beam                                               | 41       |
|         | III.4 Three Layered Beam                                                      | 41       |
|         | III.5 Five Layered Beam                                                       | 44       |
|         | III.6 Seven Layered Beam                                                      | 47       |

|           |                                                                                                           |            |
|-----------|-----------------------------------------------------------------------------------------------------------|------------|
| <b>IV</b> | <b>DAMPING CHARACTERISTICS OF MULTI-LAYERED BEAMS</b>                                                     | <b>53</b>  |
|           | <b>IV.1 Introduction</b>                                                                                  | <b>53</b>  |
|           | <b>IV.2 Reference Beam</b>                                                                                | <b>55</b>  |
|           | <b>IV.3 Constant Weight Criterion</b>                                                                     | <b>55</b>  |
|           | <b>IV.4 Constant Size Criterion</b>                                                                       | <b>55</b>  |
|           | <b>IV.5 Constant Static Stiffness Criterion</b>                                                           | <b>56</b>  |
|           | <b>IV.6 Comparative Weight, Size and Static Stiffness for various Criteria</b>                            | <b>57</b>  |
|           | <b>IV.7 System Loss Factor and Displacement Response Effectiveness</b>                                    | <b>62</b>  |
|           | <b>IV.8 Computation</b>                                                                                   | <b>69</b>  |
|           | <b>IV.9 Discussion of the Results for System Loss Factor of Multilayered Beams</b>                        | <b>76</b>  |
|           | <b>IV.10 Discussion of the Results for Displacement Response Effectiveness of Multilayered Beams</b>      | <b>93</b>  |
| <b>V</b>  | <b>RESPONSE OF DAMPED UNSYMMETRICAL MULTILAYERED BEAMS WITH END CONDITIONS OTHER THAN SIMPLE SUPPORTS</b> | <b>98</b>  |
|           | <b>V.1 Introduction</b>                                                                                   | <b>98</b>  |
|           | <b>V.2 Expression of Energy Functional for Sandwich Beam</b>                                              | <b>101</b> |
|           | <b>V.3 Boundary Conditions for Sandwich Beam</b>                                                          | <b>105</b> |
|           | <b>V.4 Solution for Fixed-Fixed Sandwich Beam</b>                                                         | <b>106</b> |

|      |                                                                                                      |     |
|------|------------------------------------------------------------------------------------------------------|-----|
| V.5  | Response of Fixed-Fixed Sandwich Beam for Displacement Excitation at ends                            | 109 |
| V.6  | Solution for Fixed-Fixed Sandwich Beam with Concentrated Mass                                        | 110 |
| V.7  | Response of Fixed-Fixed Sandwich Beam with a Concentrated Mass for Displacement Excitation at ends   | 113 |
| V.8  | Solution of Cantilever Sandwich Beam                                                                 | 113 |
| V.9  | Response of Cantilever Sandwich Beam for Displacement Excitation at Fixed End                        | 118 |
| V.10 | Solution for Cantilever Sandwich Beam with Concentrated Mass at Free End                             | 118 |
| V.11 | Response of Cantilever Sandwich Beam with Concentrated Mass for Displacement Excitation at Fixed End | 120 |
| VI   | EXPERIMENTAL WORK                                                                                    | 122 |
| VI-A | DYNAMIC SHEAR PROPERTIES OF VISCOELASTIC MATERIAL                                                    | 122 |
|      | VIA.1 Experimental Arrangement                                                                       | 123 |
|      | VIA.2 Experimental Procedure                                                                         | 129 |
|      | VIA.3 Determination of Results                                                                       | 130 |
| VI-B | VIBRATION RESPONSE TESTS ON SANDWICH BEAMS                                                           | 134 |
|      | VIB.1 Introduction                                                                                   | 134 |
|      | VIB.2 Development of Test Rig                                                                        | 136 |

|     |                                |     |
|-----|--------------------------------|-----|
|     | VIB.3 Excitation Arrangement   | 136 |
|     | (a) Fixed-Fixed Sandwich Beam  | 136 |
|     | (b) Cantilever Sandwich Beam   | 140 |
|     | VIB.4 Preparation of Specimens | 143 |
|     | VIB.5 Testing Procedures       | 147 |
|     | VIB.6 Experimental Results     | 150 |
|     | VIB.7 Theoretical Results      | 162 |
|     | VIB.8 Discussion               | 164 |
| VII | CONCLUDING REMARKS             | 167 |
|     | References                     | 172 |
|     | Appendix I                     | 179 |
|     | Appendix II                    | 185 |
|     | Appendix III                   | 189 |
|     | Appendix IV                    | 196 |