OPF FRAMEWORKS FOR DSO OPERATIONAL, COORDINATION AND MARKET MODELS

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by

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न गुरुरधिकं तत्त्वं न गुरुरधिकं तपः ।
तत्त्वज्ञानात् परं नास्ति तस्मै श्रीगुरवे नमः ॥

---- श्रीगुरु स्तोत्रम्

No theory is higher than the Master,
No spiritual practice is greater than the Master.
There is nothing beyond deep knowledge. Yet,
there is Master, whom we offer salutations.

---- Shri Guru Strotram

Dedicated to..

My beloved grandparents, Mr. Hanuman Sahai Khandelwal and Late Mrs. Radha Devi,

and my beloved parents, Mr. Om Prakash Gupta and Mrs. Shashi Khandelwal,

whose blessings and sacrifices made this journey possible.

CERTIFICATE

This is to certify that the thesis entitled **'OPF Frameworks for DSO Operational, Coordination and Market Models'**, being submitted by **Meenakshi Khandelwal** to **Indian Institute of Technology, Delhi** is a record of bonafide research work carried out under my supervision and I consider it worthy for consideration of the award of the degree **Doctor of Philosophy** in Electrical Engineering. The results obtained here have not been submitted to any other University or Institute for the award of any degree or diploma.

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Meenakshi Khandelwal

Abstract

Proliferation of distributed energy resources (DERs) in active distribution networks (ADNs) brings challenges for the distribution system operator (DSO) pertaining to their optimal scheduling, pricing, coordination among stakeholders, and market participation. This thesis addresses these critical issues by developing OPF-based optimization frameworks under different operating conditions. In this regard, a network-dependent, sensitivity-based, quadratic approximation of the branch-flow Distribution Optimal Power Flow (DOPF) model is proposed. This model is decomposed to compute active and reactive power distribution locational marginal prices (DLMPs), comprising incremental costs for energy, loss, congestion, and voltage components. These price signals vary according to the ADN operating conditions, thereby aiding DSO in incentivizing DERs for voltage support and congestion relief. Further, the proposed DOPF model is extended to incorporate the dynamic capabilities of heterogeneous DERs, enabling cost-benefit analyses for enhanced DSO operations. Moreover, the grid events arising from forecast errors and inherent uncertainties can result in load curtailment at the cost of economic losses and consumers' discomfort. To address this issue, an OPF-based electro-thermal DER scheduling framework within the integrated energy system (IES) is proposed. This supports resilient and economically efficient ADN operations, preventing it from collapsing or heavy load shedding during uncertain events.

The emergence of Distributed Energy Resource Aggregators (DERAs) to aggregate numerous small-scale DERs within ADN challenges their coordination with the DSO. To address this challenge, this thesis develops DER aggregation and DSO-DERA coordination under different market settings. A box polytope/ hyperrectangle-based geometric approach is proposed for characterizing DER flexibility. Using this method, the DERA aggregates operating regions of individual DERs through the Minkowski sum and submits a collective bid to the DSO. The DSO then performs an ADN-constrained optimization and sends disaggregated signals back to the DERA. This approach reduces the computational and communication burden of the DSO while empowering the DERA to distribute the schedules among its contracted DERs, thereby maximizing their benefits. With this approach, the DERA can operate at any point within the aggregated feasible region, ensuring compliance with ADN operational constraints. Furthermore, three DSO-DERA coordination frameworks for flexibility allocation under different market settings, i.e., joint, sequential, and independent, are proposed while ensuring ADN limits. The joint flexibility allocation (JFA) framework co-optimizes energy and flexibility markets. The sequential flexibility allocation (SFA) framework separates these markets via nodal price-based flexibility allocation, enabling flexibility evaluation by DERA. The independent flexibility allocation (IFA) framework evaluates nodal injection/withdrawal capabilities at DER nodes, facilitating autonomous DERA participation in local and wholesale markets without DSO interference.

In summary, this thesis provides a set of comprehensive tools and methodologies for the DSO and DERA to manage DERs, improving the operational and economic efficiency of ADNs. It offers scalable, market-integrated solutions that enhance DER participation in system services and electricity markets, supporting the transition toward a decentralized, resilient, and sustainable power system.

Key Words: Active Distribution Network (ADN), Distributed Energy Resources (DERs), DER Aggregator (DERA), Distribution Locational Marginal Prices (DLMPs), Distribution System Operator (DSO), Congestion, Optimal Power Flow (OPF).

सारांश

सक्रिय वितरण नेटवर्क (ADN) में वितरित ऊर्जा संसाधनों (DER) का प्रसार वितरण प्रणाली संचालक (DSO) के लिए उनके इष्टतम शेड्यूलिंग, मूल्य निर्धारण, हितधारकों के बीच समन्वय और बाजार भागीदारी से संबंधित चुनौतियाँ लाता है। यह थीसिस विभिन्न परिचालन स्थितियों के तहत OPF-आधारित अनुकूलन रूपरेखाएँ विकसित करके इन महत्वपूर्ण मुद्दों को संबोधित करती है। इस संबंध में, शाखा-प्रवाह वितरण इष्टतम पावर फ्लो (DOPF) मॉडल का एक नेटवर्क-निर्भर, संवेदनशीलता-आधारित, द्विघात सन्निकटन प्रस्तावित किया गया है। इस मॉडल को सक्रिय और प्रतिक्रियाशील बिजली वितरण स्थानीय सीमांत कीमतों (DLMP) की गणना करने के लिए विघटित किया जाता है, जिसमें ऊर्जा, हानि, भीड़ और वोल्टेज घटकों के लिए वृद्धिशील लागतें शामिल होती हैं। ये मूल्य संकेत ADN परिचालन स्थितियों के अनुसार भिन्न होते हैं, जिससे वोल्टेज समर्थन और भीड़भाड़ से राहत के लिए DER को प्रोत्साहित करने में DSO की सहायता होती है। इसके अलावा, प्रस्तावित DOPF मॉडल को विषम DER की गतिशील क्षमताओं को शामिल करने के लिए विस्तारित किया गया है, जिससे उन्नत DSO संचालन के लिए लागत-लाभ विश्लेषण सक्षम हो सके। इसके अलावा, पूर्वानुमान त्रुटियों और अंतर्निहित अनिश्चितताओं से उत्पन्न ग्रिड घटनाओं के परिणामस्वरूप आर्थिक नुकसान और उपभोक्ताओं की असुविधा की कीमत पर लोड में कटौती हो सकती है। इस मुद्दे को संबोधित करने के लिए, एकीकृत ऊर्जा प्रणाली (IES) के भीतर एक OPF-आधारित इलेक्ट्रो-थर्मल DER शेड्यूलिंग ढांचा प्रस्तावित किया गया है। यह लचीले और आर्थिक रूप से कुशल ADN परिचालनों को समर्थन प्रदान करता है, तथा अनिश्चित घटनाओं के दौरान इसके ध्वस्त होने या भारी लोड शेडिंग से बचाता है।

ADN के भीतर कई छोटे पैमाने के DER को एकत्रित करने के लिए वितरित ऊर्जा संसाधन एग्रीगेटर (DERA) का उद्भव DSO के साथ उनके समन्वय को चुनौती देता है। इस चुनौती का समाधान करने के लिए, यह थीसिस विभिन्न बाजार सेटिंग्स के तहत DER एकत्रीकरण और DSO-DERA समन्वय विकसित करती है। DER लचीलेपन को चिह्नित करने के लिए एक बॉक्स पॉलीटॉप/हाइपररेक्टेंगल-आधारित ज्यामितीय दृष्टिकोण प्रस्तावित किया गया है। इस पद्धति का उपयोग करते हुए, DERA मिंकोव्स्की योग के माध्यम से व्यक्तिगत DER के संचालन क्षेत्रों को एकत्रित

करता है और DSO को एक सामूहिक बोली प्रस्तुत करता है। DSO तब ADN-प्रतिबंधित अनुकूलन करता है और अलग-अलग संकेतों को DERA को वापस भेजता है। यह दृष्टिकोण DSO के कम्प्यूटेशनल और संचार बोझ को कम करता है जबकि DERA को अपने अनुबंधित DER के बीच अनुसूचियों को वितरित करने के लिए सशक्त बनाता है, जिससे उनके लाभ अधिकतम हो जाते हैं। इस दृष्टिकोण के साथ, DERA एकत्रित व्यवहार्य क्षेत्र के भीतर किसी भी बिंदु पर काम कर सकता है, जिससे ADN परिचालन बाधाओं का अनुपालन सुनिश्चित होता है। इसके अलावा, विभिन्न बाजार सेटिंग्स के तहत लचीलेपन के आवंटन के लिए तीन DSO-DERA समन्वय ढांचे, यानी संयुक्त, अनुक्रमिक और स्वतंत्र, ADN सीमाओं को सुनिश्चित करते हुए प्रस्तावित किए गए हैं। संयुक्त लचीलापन आवंटन (JFA) ढांचा ऊर्जा और लचीलेपन के बाजारों को सह-अनुकूलित करता है। अनुक्रमिक लचीलापन आवंटन (SFA) ढांचा नोडल मूल्य-आधारित लचीलेपन के आवंटन के माध्यम से इन बाजारों को अलग करता है, जिससे DERA द्वारा लचीलेपन का मूल्यांकन संभव हो जाता है। स्वतंत्र लचीलापन आवंटन (IFA) ढांचा DER नोड्स पर नोडल इंजेक्शन/निकासी क्षमताओं का मूल्यांकन करता है, जिससे DSO के हस्तक्षेप के बिना स्थानीय और थोक बाजारों में स्वायत्त DERA भागीदारी की सुविधा मिलती है।

संक्षेप में, यह थीसिस डीएसओ और डीईआरए को डीईआर का प्रबंधन करने के लिए व्यापक उपकरणों और पद्धतियों का एक सेट प्रदान करती है, जिससे एडीएन की परिचालन और आर्थिक दक्षता में सुधार होता है। यह स्केलेबल, बाजार-एकीकृत समाधान प्रदान करता है जो सिस्टम सेवाओं और बिजली बाजारों में डीईआर की भागीदारी को बढ़ाता है, एक विकेन्द्रीकृत, लचीले और टिकाऊ बिजली प्रणाली की ओर संक्रमण का समर्थन करता है।

प्रमुख शब्द: सक्रिय वितरण नेटवर्क (ADN), वितरित ऊर्जा संसाधन (DERs), DER एग्रीगेटर (DERA), वितरण स्थानिक सीमांत मूल्य (DLMPs), वितरण प्रणाली ऑपरेटर (DSO), भीड़भाड़, इष्टतम विद्युत प्रवाह (OPF)।

Contents

List of Figures

List of Tables

Nomenclature

A. Acronyms/Abbreviations

ACOPF	Alternating Current Optimal Power Flow
ADN	Active Distribution Network
BESS	Battery Energy Storage System
CEA	Central Electricity Authority
CERC	Central Electricity Regulatory Commission
CHP	Combined Heat and Power
CS-OPF	Current Sensitivity-based Optimal Power Flow
DCOPF	Direct Current Optimal Power Flow
DERA	Distributed Energy Resource Aggregator
DERs	Distributed Energy Resources
DG	Dispatchable Generator
DLMP	Distribution Locational Marginal Price
DN	Distribution Network

DOPF	Distributed Optimal Power Flow
DR	Demand Response
DSO	Distribution System Operator
EH	Energy Hub
EVs	Electric Vehicles
FLs	Flexible Loads
GAMS	General Algebraic Modeling System
IES	Integrated Energy System
IFA	Independent Flexibility Allocation
IOPF-C	Iterative OPF with loss as fictitious nodal demand
IOPF-J	Loss sensitivity-based iterative OPF
JFA	Joint Flexibility Allocation
LEM	Local Energy Market
LFM	Local Flexibility Market
LHV	Lower Heating Value
LMP	Locational Marginal Price
LOPF	Linearized Optimal Power Flow
MAE	Mean Absolute Error
MATLAB	Matrix Laboratory

MES	Multi-Energy System
OPF	Optimal Power Flow
P2P	Peer-to-Peer
RES	Renewable Energy Source
SDP	Semi-definite Programming
SFA	Sequential Flexibility Allocation
SOCP	Second-Order Cone Programming
TSO	Transmission System Operator

B. Indices and Sets

$\mathcal{A}$	Set of DERAs.
$\mathcal{CHP}$	Set of CHP with boiler units.
$\mathcal{DG}$	Set of DG units.
$\mathcal{FL}$	Set of flexible loads.
$\mathcal{G}$	Set of DER units.
$\mathcal{H}/S$	Set of thermal/electrical BESS units.
$\mathcal{L} \in (i, k)$	Set of lines/branches connected between nodes i and k.
$\mathcal{N}_{\mathcal{A}}$	Set of DER nodes aggregated by DERA.
$\mathcal{N}_{\mathcal{B}}$	Set of ADN nodes/buses.
$\mathcal{N}_{\mathcal{F}}$	Set of flexible loads.

$\mathcal{N}_{\mathcal{G}}$	Set of DERs aggregated by DERA.
$\mathcal{T}$	Set of time intervals.
a	Index of DERA.
chp	Index of CHP units with boiler.
dg	Index of DG units.
f	Index of flexible loads.
g	Index of DER units.
h/s	Index of thermal/electrical BESS units.
i, k, m	Indices of ADN nodes/buses.
t	Index of time intervals.

C. System Parameters and Constants

η_s^{ch}/η_s^{dis}	Charging/Discharging efficiency of electrical BESS unit s.
$\eta_{ee}^{chp}/\eta_{bl}^{chp}$	Electrical/Thermal power conversion efficiency of CHP with boiler BESS unit chp.
η_h^{ch}/η_h^{dis}	Charging/Discharging efficiency of thermal BESS unit h.
$\overline{P}_{chp}^{ee}$	Maximum electrical power generation by CHP with boiler unit chp [kW].
$\overline{P}_h^{ch}/\overline{P}_h^{dis}$	Maximum charging/discharging limit of thermal BESS unit h [kW].
$\overline{P}_s^{ch}/\overline{P}_s^{dis}$	Maximum charging/discharging limit of electrical BESS unit s [kW].
$\overline{R}_i$	Maximum ramp capacity at node i at time t [kW/hr].

ϕ_{chp}	Heat-to-power generation ratio of CHP unit chp.
$\tilde{P}_{gt}$	Active power schedule (by DSO) of DER g at time t [kW].
$\underline{E}_h/\overline{E}_h$	Minimum/Maximum energy limits of thermal BESS unit h [kWh].
$\underline{E}_s/\overline{E}_s$	Minimum/Maximum limits of energy stored in electrical BESS unit s [kWh].
$\underline{P}^{bl}_{chp}/\overline{P}^{bl}_{chp}$	Minimum/Maximum limit on thermal power generation by CHP with boiler unit chp [kW].
$\underline{Q}_{dg}/\overline{Q}_{dg}$	Minimum/Maximum limit on reactive power generation by DG unit dg [kVAr].
$\underline{Q}_g/\overline{Q}_g$	Minimum/Maximum limit on reactive power generation by DER unit g [kVAr].
$\underline{V}_i/\overline{V}_i$	Minimum/Maximum limit on voltage magnitude at node i [pu].
C^{A}_{at}	Active power cost bid of DERA a at time t [\$/kWh].
C^{P}_{dgt}/C^{Q}_{dgt}	Active/Reactive power generation cost of DG unit dg at time t [\$/kWh].
C^{P}_{gt}/C^{Q}_{gt}	Active/Reactive power bid of DER g at time t [\$/kWh].
C^{PoC}_{t}	Active power price at PoC, time t [\$/kWh].
F^{uR}_{t}/F^{dR}_{t}	Total upward/downward flexibility request (by DSO) at time t [kW].
$H_{km,i}$	Network incidence matrix with PoC as reference node $[\mathcal{N}_{\mathcal{L}} \times \mathcal{N}_{\mathcal{B}}]$.
P^{D}_{it}/Q^{D}_{it}	Active/Reactive power demand at node i, time t [kW].
T^{D}_{it}	Thermal power demand at node i, time t [kW].
$VoLL$	Value of lost load [\$/kWh].
$\overline{I}^{'}_{ik}$	Maximum squared current capacity of line connected between nodes i and k [pu].

$\overline{R}^{G}_{gt}$	Maximum ramp capacity of DER g at time t [kW/hr].
$\overline{V}^{'}_{i}$	Squared maximum voltage magnitude limit at node i [pu].
$\tilde{P}l_{kjt}/\tilde{P}u_{kjt}$	Minimum/maximum capabilities of active power flow (after energy schedule) at the line connected between nodes k and j, at time t [kW].
$\underline{P}^{A}_{at}/\overline{P}^{A}_{at}$	Minimum/Maximum active power bid by DERA a at time t [kW].
$\underline{P}^{inj}_{it}/\overline{P}^{inj}_{m}$	Minimum/Maximum active power injection capacity at node i at time t [kW].
$\underline{P}_{dg}/\overline{P}_{dg}$	Minimum/Maximum limit on active power generation of DG unit dg [kW].
$\underline{P}_{g}/\overline{P}_{g}$	Minimum/Maximum limit on active power generation of DER unit g [kW].
$\underline{P}_{ik}/\overline{P}_{ik}$	Minimum/Maximum limit on active power flow at line connected between nodes i and k [kW].
$\underline{V}'_{i}/\overline{V}^{'}_{i}$	Minimum/Maximum limit on squared voltage magnitude at bus i [pu].
$C^{bl}_{chp}/C^{ee}_{chp}$	Cost of heat/electricity generation by CHP with boiler unit *chp* [\$/kWh].
C_{gh}/C_{gg}	Fuel cost of heat/electricity generation by CHP with boiler unit *chp* [\$/kg].

D. Decision Variables

δ_{it}	Voltage angle at node i, time t [rad].
C_{chpt}	Total cost of CHP with boiler unit *chp* at time t [\$].
C_{dgt}	Total cost of DG unit dg at time t [\$].
C_{ft}	Total cost of flexible load f at time t [\$].
E_{ht}	Energy stored in thermal BESS unit h at time t [kWh].
E_{st}	Energy stored in electrical BESS unit s at time t [kWh].

F^{u}_{gt}/F^{d}_{gt}	Upward/downward active power flexibility capabilities (after energy schedule) of DER g at time t [kW].
F^{un}_{gt}/F^{dn}_{gt}	Upward/downward flexibilities allocated (upon DSO request) to DER g at time t [kW].
F^{ur}_{gt}/F^{dr}_{gt}	Upward/downward ramp capabilities (after energy schedule) of DER g at time t [kW/hr].
$I^{'}_{ikt}$	Squared line current flowing between nodes i and k at time t [pu].
P^{A}_{at}	Active power schedule of DERA a at time t [\$/kWh].
P^{bl}_{chpt}	Thermal power output from CHP with boiler unit *chp* at time t [kW].
P^{ee}_{chpt}	Electrical power output from CHP with boiler unit *chp*, time t [kW].
P^{inj}_{m}	Active power injection at node m [kW].
P_{dgt}/Q_{dgt}	Active/Reactive power schedule of DG unit dg at time t [kW/kVAr].
P_{ft}	Flexible load f at time t [kW].
P_{gt}/Q_{gt}	Active/Reactive power schedule of DER g at time t [kW/kVAr].
P^{ch}_{ht}/P^{dis}_{ht}	Charging/Discharging power drawn/supplied by thermal BESS unit h at time t [kW].
P_{ikt}/Q_{ikt}	Active/Reactive power flow on line connected between nodes i and k at time t [kW/kVAr].
P^{L}_{ikt}/Q^{L}_{ikt}	Active/Reactive power loss of line connected between nodes i and k at time t [kW/kVAr].

P_{st}^{ch}/P_{st}^{dis}	Charging/Discharging power drawn/supplied by electrical BESS unit s at time t [kW].
Pl_{ikt}/Pu_{ikt}	Lower/upper active power flow capability (after energy schedule) of line connected between nodes i and j at time t [kW].
$Q_{dgt}^{'}$	Linear variable associated with reactive power generation from DG unit dg [kVAr].
$Q_{gt}^{'}$	Linearized reactive power variable of DER g at time t [kVAr].
$V_{it}^{'}$	Squared voltage magnitude at node i, time t [pu].

E. Lagrangian Variables and Sensitivities

β_1/β_2	Lagrangian variable associated with active/reactive power balance.
β_5/β_6	Lagrangian variable associated with active/reactive power loss.
γ_7/γ_8	Lagrangian variable associated with lower/upper limits of voltage magnitude.
γ_9	Lagrangian variable associated with branch flow current limits.
$\lambda_{it}^{P}/\lambda_{it}^{Q}$	Active/Reactive power DLMP at node i, time t.
S_{fp}	Sensitivity of line current with respect to active power injection.
S_{fq}	Sensitivity of line current with respect to reactive power injection.
S_{pp}	Sensitivity of active power loss with respect to active power injection.
S_{pq}	Sensitivity of active power loss with respect to reactive power injection.
S_{qp}	Sensitivity of reactive power loss with respect to active power injection.
S_{qq}	Sensitivity of reactive power loss with respect to reactive power injection.

S_{vp}	Sensitivity of voltage magnitude with respect to active power injection.
S_{vq}	Sensitivity of voltage magnitude with respect to reactive power injection.