

**AN EFFICIENT LAYERWISE MULTI-PHYSICS
SPECTRAL ELEMENT MODEL FOR GUIDED
WAVES IN SMART STRIPS WITH
DELAMINATIONS**

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SMART STRIPS WITH DELAMINATIONS

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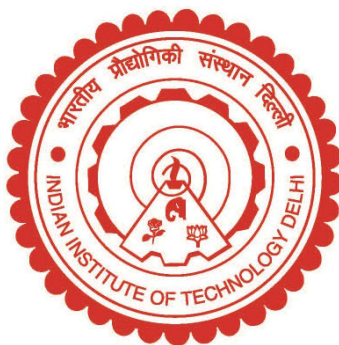
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Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



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*Dedicated to
my mother*

Certificate

This is to certify that the thesis entitled “**An Efficient Layerwise Multi-Physics Spectral Element Model for Guided Waves in Smart Strips with Delaminations**” being submitted by **Mr. Mayank Jain** to the Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

Fibre-reinforced composite laminates are increasingly used in high performance structures such as aircraft, automobiles, spacecraft, propellers, spaceships, wind turbine blades, bridges, sports and medical equipments. However, their significant anisotropic and inhomogeneous characteristics make them prone to a rather serious mode of invisible sub-surface damage, namely delamination at ply interfaces. Thus, a structural health monitoring (SHM) strategy becomes imperative for ensuring the integrity and reliability of such laminated composite structures. Lamb waves are often used for both off-line non-destructive evaluation (NDE) and online SHM for identifying the sub-surface defects in thin-walled structures. Piezoelectric wafer active sensors (PWAS) transducer patches, surface-bonded or embedded in the laminate, provide a very low cost and convenient option for actuation and sensing of high-frequency guided waves. Both model-assisted and physics-informed data-driven strategies for SHM require solving the inverse problem involving solutions of a large number of forward wave propagation problems. Thus, there is a need to develop computational models that can predict guided wave propagation in smart laminated structures accurately and at the same time efficiently.

In this thesis, a novel computationally efficient multiphysics time-domain spectral element (TDSE) model is presented for simulating Lamb waves in smart composite strips (beams and panels) integrated with surface-bonded or embedded patch or full layers of PWAS transducers, featuring delaminations in the host laminate. The delaminations are modelled using the region approach based on the free mode assumption. A new C^1 -continuous spectral interpolation using the Gauss–Lobatto–Legendre basis is proposed, which is shown to eliminate the Runge phenomenon observed in the conventional higher-order Hermite interpolation. This interpolation function is used in an efficient layerwise zigzag theory (ZIGT) for kinematics, which combines with a piecewise quadratic variation for the electric potential field to develop an electromechanically coupled formulation for modelling intact host laminate, patch transducer-bonded laminates, and sub-laminates between adjacent delamination. The normal deformation

caused by the electric field is incorporated in the deflection without introducing additional displacement variables beyond the smeared third-order theory (TOT). The high-order multifield TDSE based on this coupled ZIGT has n physical nodes containing mechanical and electric degrees of freedom (DOFs) and a separate virtual electric node for modelling the transducers' equipotential surfaces with significantly reduced DOFs. The recently proposed hybrid point-least squares continuity (HPLSC) method is adapted for the spectral strip element to satisfy the continuity of the nonlinear in-plane displacement field across the laminate thickness at the delamination fronts. These conditions are imposed using a transformation matrix that relates the nodal DOFs of the sub-laminate at the delamination front to those of the intact laminate. In addition, the segment integrated with PWAS is connected to the host segment via the patch fronts at which the continuity is maintained using the same advanced HPLSC model. The methodology offers a general framework for modelling multiple delaminations and surface-bonded or embedded transducer patches at arbitrary interfacial or longitudinal locations. The coupled semi-discrete governing equations of motion for the piezolaminated system are derived using the Hamilton's principle.

The developed spectral element is validated and critically assessed on the basis of accuracy, efficiency, and convergence through a comprehensive numerical study for static, free vibration, and wave propagation analyses of elastic and piezoelectric smart composite strips, with and without delaminations. The Lamb wave propagation examples are selected so as to represent actual SHM applications. The present results are validated using experiments on the actuation and sensing of guided waves and compared to the previously published data, analytical ZIGT solutions, continuum-based finite element solutions from ABAQUS, and different laminate theory-based solutions. It is shown that the present element yields excellent accuracy with much faster convergence, higher computational efficiency and many-fold reduction in computational time than its conventional finite element counterpart for narrowband high frequency guided wave propagation problems in both undamaged and damaged strips. The role of the slope discontinuities in the in-plane displacement approximation is delineated by comparing it with the smeared TOT-based SE solution with the same number of DOFs. The effect of imposing continuity at the patch fronts is investigated for free vibration and wave propagation responses of the smart strips. The developed model is used to study the interaction of Lamb waves with notch-type damages. Finally, the effect of delamination on sensor signal due to Lamb wave propagation in smart composite strips is also studied to explore the identification of interfacial location and sizing of delamination damage. This development will directly benefit the model-based and physics-informed data-driven SHM of composite structures.

सारांश

फाइबर-प्रबलित कम्पोजिट लैमिनेट्स का उपयोग विमान, ऑटोमोबाइल, अंतरिक्ष यान, प्रोपेलर, पवन टरबाइन ब्लेड, पुल, खेल और चिकित्सा उपकरणों जैसे उच्च प्रदर्शन संरचनाओं में तेजी से किया जा रहा है। हालाँकि, उनकी महत्वपूर्ण विषमदैशिक और विषमरूपी विशेषताएँ उन्हें अदृश्य उप-सतह क्षति, अर्थात् प्लाई इंटरफेस पर डीलेमिनेशन, के लिए एक गंभीर तरीके से ग्रस्त बनाती हैं। इस प्रकार, ऐसी लेमिनेटेड समग्र संरचनाओं की अखंडता और विश्वसनीयता सुनिश्चित करने के लिए एक संरचनात्मक स्वास्थ्य निगरानी (एसएचएम) रणनीति अनिवार्य हो जाती है। पतली दीवार वाली संरचनाओं में उप-सतह क्षति की पहचान करने के लिए लैम्ब तरंगों का उपयोग अक्सर ऑफ़-लाइन गैर-विनाशकारी मूल्यांकन (एनडीई) और ऑनलाइन एसएचएम दोनों के लिए किया जाता है। पीज़ोइलेक्ट्रिक वेफर सक्रिय सेंसर (पीडब्ल्यूएस) ट्रांसड्यूसर पैच, सतह से जुड़े या लेमिनेट में एम्बेडेड, उच्च आवृत्ति निर्देशित तरंगों के क्रियान्वयन और संवेदन के लिए बहुत कम लागत और सुविधाजनक विकल्प प्रदान करते हैं। एसएचएम के लिए दोनों रणनीतियों, मॉडल-सहायता प्राप्त और भौतिकी-सूचित डेटा-संचालित, को इनवर्स प्रॉब्लम को हल करने की आवश्यकता होती है जिसमें बड़ी संख्या में फॉरवर्ड तरंग प्रसार समस्याओं का समाधान शामिल होता है। इस प्रकार, कम्प्यूटेशनल मॉडल विकसित करने की आवश्यकता है जो स्मार्ट लेमिनेटेड संरचनाओं में निर्देशित तरंग प्रसार की सटीक और साथ ही कुशलतापूर्वक भविष्यवाणी कर सके।

इस थीसिस में, सतह से जुड़े या लेमिनेट में एम्बेडेड पैच या पीडब्ल्यूएस ट्रांसड्यूसर की पूरी परतों के साथ एकीकृत स्मार्ट कम्पोजिट स्ट्रिप्स (बीम और पैनल), जिसमें डीलेमिनेशन होस्ट लैमिनेट में शामिल है, में लैम्ब तरंगों को अनुकरण करने के लिए एक नव कम्प्यूटेशनल रूप से कुशल मल्टीफिज़िक्स टाइम-डोमेन स्पेक्ट्रल एलिमेंट (टीडीएसई) मॉडल प्रस्तुत किया गया है। मुक्त मोड धारणा के आधार पर क्षेत्र दृष्टिकोण का उपयोग करके डीलेमिनेशन को मॉडल किया गया है। गॉस-लोबेटो-लेजेंड्रे के आधार पर एक नया C^1 -निरंतर स्पेक्ट्रल अंतरवेषण प्रस्तावित है, जो पारंपरिक उच्च-क्रम हर्माइट अंतरवेषण में देखी गई रंगे घटना को दूर करने के लिए दिखाया गया है। इस अंतरवेषण फलन का उपयोग किनेमेटिक्स के लिए एक कुशल लेयरवाइज़ ज़िगज़ैग सिद्धांत (ज़िगटी) में किया जाता है, जो विद्युत संभावित क्षेत्र के लिए एक खंड अनुसार द्विघात भिन्नता के साथ मिलकर अक्षुण्ण होस्ट लैमिनेट, पैच ट्रांसड्यूसर-बोंडेड लैमिनेट्स और आसन्न डीलेमिनेशन के बीच सब-लेमिनेट्स के मॉडलिंग के लिए एक इलेक्ट्रोमैकेनिकल रूप से युग्मित सूत्रीकरण विकसित करता है। विद्युत क्षेत्र के कारण होने वाली नार्मल विकृति को स्मिथर्ड थर्ड-ऑर्डर सिद्धांत (टीओटी) से परे अतिरिक्त विस्थापन चर पेश किए बिना विचलन में शामिल किया गया है। इस युग्मित ज़िगटी पर आधारित उच्च-क्रम बहु-क्षेत्र टीडीएसई में n भौतिक नोड्स हैं जिनमें मैकेनिकल और इलेक्ट्रिक डिग्री ऑफ़ फ्रीडम (डीओएफ) शामिल हैं और ट्रांसड्यूसर की समविभव सतहों को काफी कम डीओएफ के

साथ मॉडलिंग करने के लिए एक अलग वर्चुअल इलेक्ट्रिक नोड है। हाल ही में प्रस्तावित हाइब्रिड पॉइंट-लीस्ट स्क्रायर कॉन्टिनुइटी (एचपीएलएससी) विधि को स्पेक्ट्रल स्ट्रिप एलिमेंट के लिए अनुकूलित किया गया है ताकि डीलेमिनेशन प्रतिच्छेदन पर लेमिनेट की मोटाई में अरेखीय इन-प्लेन विस्थापन क्षेत्र की निरंतरता को संतुष्ट किया जा सके। इन शर्तों को एक परिवर्तन मैट्रिक्स का उपयोग करके लगाया जाता है जो डीलेमिनेशन के प्रतिच्छेदन पर सब-लेमिनेट के नोडल डीओएफ को अक्षुण्ण लैमिनेट के नोडल डीओएफ से संबंधित करता है। इसके अलावा, पीडब्ल्यूएस के साथ एकीकृत खंड पैच प्रतिच्छेदन के माध्यम से होस्ट खंड से जुड़ा हुआ है, जिस पर समान उन्नत एचपीएलएससी मॉडल का उपयोग करके निरंतरता बनाए रखी जाती है। यह पद्धति किसी भी इंटरफेसियल या अक्षीय स्थानों पर एकाधिक डीलेमिनेशन और सतह से जुड़े या लेमिनेट में एम्बेडेड ट्रांसड्यूसर पैच के मॉडलिंग के लिए एक सामान्य रूपरेखा प्रदान करती है। पीज़ोलैमिनेटेड प्रणाली के लिए गति के युग्मित अर्ध-असतत संचालक समीकरण हैमिल्टन के सिद्धांत का उपयोग करके प्राप्त किए गए हैं।

विकसित स्पेक्ट्रल एलिमेंट को स्थैतिक, मुक्त कंपन और तरंग प्रसार विश्लेषण के लिए एक व्यापक संख्यात्मक अध्ययन के माध्यम से सटीकता, दक्षता और अभिसरण के आधार पर प्रत्यास्थ और पीजोइलेक्ट्रिक स्मार्ट कम्पोजिट स्ट्रिप्स (साथ और बिना डीलेमिनेशन) को मान्य और गंभीर रूप से मूल्यांकन किया गया है। वास्तविक एसएचएम अनुप्रयोगों का प्रतिनिधित्व करने के लिए लैम्ब तरंग प्रसार उदाहरणों का चयन किया गया है। वर्तमान परिणामों को निर्देशित तरंगों के सक्रियण और संवेदन पर प्रयोगों का उपयोग करके मान्य किया गया है और पहले प्रकाशित डेटा, विश्लेषणात्मक ज़िगटी समाधान, ABAQUS से सातत्य-आधारित फाइनाइट एलिमेंट समाधान और विभिन्न लेमिनेट सिद्धांत-आधारित समाधानों से तुलना की गयी है। यह दिखाया गया है कि वर्तमान एलिमेंट अक्षतिग्रस्त और क्षतिग्रस्त दोनों स्ट्रिप्स में नैरोबैंड उच्च आवृत्ति निर्देशित तरंग प्रसार समस्याओं के लिए अपने समकक्ष पारंपरिक फाइनाइट एलिमेंट की तुलना में बहुत तेज अभिसरण, उच्च कम्प्यूटेशनल दक्षता और कम्प्यूटेशनल समय में कई गुना कमी के साथ उत्कृष्ट सटीकता प्रदान करता है। इन-प्लेन विस्थापन सन्निकटन में ढलान अनिरंतरता की भूमिका को डीओएफ की समान संख्या में स्मियर्ड टीओटी-आधारित एसई समाधान के साथ तुलना करके वर्णन किया गया है। स्मार्ट स्ट्रिप्स के मुक्त कंपन और तरंग प्रसार प्रतिक्रियाओं के लिए पैच प्रतिच्छेदन पर निरंतरता लागू करने के प्रभाव की जांच की जाती है। विकसित मॉडल का उपयोग नॉच-प्रकार की क्षति के साथ लैम्ब तरंगों की परस्पर क्रिया का अध्ययन करने के लिए किया गया है। अंत में, डीलेमिनेशन के इंटरफेसियल स्थान की पहचान और उसके आकार का पता लगाने के लिए स्मार्ट कम्पोजिट स्ट्रिप्स में लैम्ब तरंग प्रसार के कारण सेंसर सिग्नल पर डीलेमिनेशन के प्रभाव का भी अध्ययन किया गया है। इस विकास से समग्र संरचनाओं के मॉडल-आधारित और भौतिकी-सूचित डेटा-संचालित एसएचएम को सीधे लाभ होगा।

Contents

Certificate	i
Acknowledgements	ii
Abstract	iii
Contents	vii
List of Figures	xiii
List of Tables	xxv
List of Abbreviations	xxviii
1 Introduction	1
1.1 PREFACE	1
1.2 LITERATURE REVIEW	3
1.2.1 Wave propagation analysis of isotropic structures	4
1.2.2 Wave propagation analysis of composite structures	8
1.2.3 Wave propagation analysis of delaminated composite structures	12
1.2.4 Wave propagation analysis of piezolaminated smart structures	16

1.2.5	Wave propagation analysis of piezolaminated structures featuring delaminations	19
1.3	MOTIVATION	22
1.4	OBJECTIVES	23
1.5	ORGANISATION OF THE THESIS	24
2	C¹-Continuous Time-Domain Spectral Element for Euler-Bernoulli Beams	28
2.1	INTRODUCTION	28
2.2	HAMILTON'S PRINCIPLE FOR EULER-BERNOULLI BEAM	29
2.3	C ¹ -CONTINUOUS SPECTRAL FINITE ELEMENT FORMULATION	30
2.3.1	Lobatto interpolation for spectral element	30
2.3.2	Generalised Hermite interpolation for n -node element	33
2.3.3	Semi-discretized equations of motion	35
2.3.4	Lobatto integration quadrature for computation of element matrices	38
2.4	SOLUTION OF SYSTEM EQUATION OF MOTION	39
2.4.1	Direct explicit method (DEM)	39
2.4.2	Guyan reduction method (GRM)	40
2.5	NUMERICAL RESULTS AND DISCUSSION	41
2.5.1	Free vibration analysis	41
2.5.2	Wave propagation in beam under tone burst excitation	42
2.5.3	Wave propagation in notched beam	49
2.5.4	Wave propagation in beam under initial displacement	52
2.5.5	Wave propagation response under impact load	53
2.6	CONCLUSIONS	55

3	Refined TOT-based Spectral Element for Isotropic and Composite Strips	59
3.1	INTRODUCTION	59
3.2	REFINED THIRD-ORDER SHEAR DEFORMABLE THEORY	60
3.2.1	Displacement field approximations	62
3.2.2	Weak formulation of the governing equation	63
3.2.3	Displacement formulation	64
3.3	SPECTRAL INTERPOLATION FOR n -NODE ELEMENT	65
3.3.1	C^0 -continuous interpolation of u_0 and ψ_0	66
3.3.2	C^1 -continuous spectral interpolation of w_0	67
3.4	EQUATIONS OF MOTION FOR THE DISCRETIZED SYSTEM	67
3.5	LOBATTO INTEGRATION QUADRATURE	69
3.6	ANALYSIS OF ISOTROPIC STRIPS	71
3.6.1	Free vibration analysis	71
3.6.2	Lamb wave propagation problem-1	72
3.6.3	Lamb wave propagation problem-2	75
3.6.4	Comparison of SE with conventional FE	78
3.6.5	Convergence study: under-integration versus full integration	82
3.6.6	Effect of n on accuracy	85
3.6.7	Wave propagation under impact load	87
3.7	ANALYSIS OF LAMINATED COMPOSITE STRIPS	89
3.7.1	Free vibration analysis	89
3.7.2	Lamb wave propagation under transverse loading	92
3.7.3	Lamb wave propagation due to surface-bonded transducer actuation	94

3.7.4	Comparison of SE with conventional FE	99
3.7.5	Convergence study	101
3.7.6	Effect of n on accuracy	103
3.7.7	Wave propagation under impact load	103
3.8	CONCLUSIONS	104
4	Efficient Layerwise Spectral Element Model for Delaminated Composite Strips with PWAS Transducers	109
4.1	INTRODUCTION	109
4.2	ZIGZAG KINEMATICS OF PIEZO-LAMINATED COMPOSITE STRIPS WITH MULTIPLE DELAMINATIONS	110
4.2.1	Constitutive equations	113
4.2.2	Electric potential field approximation	114
4.2.3	Displacement field approximations	115
4.2.4	Weak formulation via Hamilton's variational principle	118
4.3	SPECTRAL ELEMENT WITH PHYSICAL AND ELECTRIC NODES	120
4.4	CONTINUITY AT DELAMINATION FRONTS	123
4.4.1	Point continuity method	125
4.4.2	Hybrid point-least squares method	126
4.5	CONTINUITY AT PATCH FRONTS	128
4.6	SEMI-DISCRETE EQUATION OF MOTION	130
4.7	CLOSURE	134
5	Wave Propagation Response of Intact and Delaminated Elastic Composite Strips	135

5.1	INTRODUCTION	135
5.2	ANALYSIS OF INTACT LAMINATED STRIPS	136
5.2.1	Free vibration response	136
5.2.2	Transversely excited Lamb wave propagation	138
5.2.3	Lamb waves generated by surface-bonded transducer in a single-material composite strip	140
5.2.4	Lamb wave propagation in an FML plate strip	143
5.2.5	Comparison of spectral element with standard finite element	145
5.3	ANALYSIS OF LAMINATED STRIPS FEATURING DELAMINATIONS . . .	150
5.3.1	Free vibration of composite beams with multiple delaminations	150
5.3.2	Lamb wave propagation problem of Ramadas et al. (2010)	153
5.3.3	Surface-bonded transducer induced Lamb wave propagation for SHM . .	157
5.3.4	Spectral element versus standard FE	160
5.3.5	Localization of delamination damage using Lamb wave response	161
5.4	CONCLUSIONS	165
6	Wave Propagation Response of Piezolaminated Composite Strips with and without Delaminations	168
6.1	INTRODUCTION	168
6.2	ANALYSIS OF INTACT PIEZO PATCH-BONDED COMPOSITE STRIPS . .	169
6.2.1	Static analysis under electromechanical loading	169
6.2.2	Free vibration analysis	174
6.2.3	Lamb wave propagation	175
6.3	ANALYSIS OF SMART STRIPS FEATURING DELAMINATIONS	189

6.3.1	Lamb wave propagation: Validation	189
6.3.2	Lamb wave propagation: Further assessment	192
6.3.3	Comparison with the standard FE	193
6.3.4	Impact of interfacial location and size of delamination on the sensor response	196
6.4	CONCLUSIONS	197
7	Conclusions	201
7.1	CONCLUSIONS FROM THE PRESENT RESEARCH	201
7.2	FUTURE SCOPE OF WORK	206
	References	207
	List of Publications from the Thesis	234
	Brief Biodata of the Author	235

List of Figures

1.1	(a) Regions containing intact laminates I_i and sub-laminates D_i separated by delaminated interfaces around a delamination zone. (b) Finite element model of these regions using two-node beam elements for each sub-laminate with common nodes at delamination fronts.	14
2.1	An n -node spectral beam element	34
2.2	C^1 -continuous shape functions for deflection DOFs ($\bar{N}_1, \bar{N}_3, \bar{N}_5, \dots, \bar{N}_{19}$) for a 10-node element with (a) uniformly spaced nodes for conventional element showing Runge oscillations near the two ends, and (b) Lobatto interpolation nodes for spectral element showing absence of Runge oscillations	36
2.3	C^1 -continuous shape functions for rotational DOFs ($\bar{N}_2, \bar{N}_4, \bar{N}_6, \dots, \bar{N}_{20}$) for a 10-node element with (a) uniformly spaced nodes for conventional element showing Runge oscillations near the two ends, and (b) Lobatto interpolation nodes for spectral element showing absence of Runge oscillations	37
2.4	Cantilever beam for free vibration analysis	41
2.5	Configuration of undamaged beam considered for wave propagation analysis showing pin-force model actuation at transducer edges B and C, and sensing location S	44
2.6	Hann window modulated ten-cycle sinusoidal tone burst signal of 100 kHz central frequency	44
2.7	Hilbert envelope of wave response used for computation of its peak amplitude and time of arrival	46

2.8	Axial strain-time history at sensor location on the top surface of undamaged beam under ten-cycle tone burst excitation using the direct explicit method for excitation frequencies: a) 100 kHz, b) 200 kHz and c) 400 kHz	47
2.9	Axial strain-time history at sensor location on the top surface of undamaged beam under ten-cycle tone burst excitation using the Guyan reduction method for excitation frequencies: a) 100 kHz, b) 200 kHz and c) 400 kHz	48
2.10	Convergence of peak amplitude (PA) and time of arrival (ToA) using Guyan reduction method for wave propagation in undamaged beam for excitation frequencies: a) 100 kHz, b) 200 kHz and c) 400 kHz	50
2.11	Configuration of beam with notch-type damage considered for wave propagation analysis	51
2.12	Axial strain-time history at sensor location on the top surface of beam with notch-type damage under 400 kHz ten-cycle tone burst excitation	51
2.13	Initial deflection profile of simply supported beam	53
2.14	Wave profiles due to initial displacement at different time instances	54
2.15	Input impulse load on the tip of the beam	55
2.16	Transverse velocity at the tip of a cantilever beam under an impact excitation without damping	56
2.17	Transverse velocity at the tip of a cantilever beam under an impact excitation with 0.01% modal damping	56
3.1	Configuration of multilayered composite strip, showing structural axes, reference plane ($z = 0$) and layer numbering. The $z = 0$ plane passes through or is at the bottom of the k_0 th layer.	60
3.2	TOT-based spectral strip element having n internal nodes showing nodal DOFs. The nodes are located at GLL points and have unequal spacing.	65
3.3	Natural frequencies of first 150 modes of the simply-supported aluminium beam. Numbers in bracket indicate total number of DOFs.	71

LIST OF FIGURES

3.4	Configuration of the plate considered for the wave propagation problem of Ge et al. (2014), showing force application in (a) 2D model and (b) 1D models. S denotes the sensing location	73
3.5	(a) Hann window modulated 5-cycle sinusoidal tone burst signal of 100 kHz central frequency. (b) Frequency spectrum of the signal	73
3.6	Time histories of (a) in-plane displacement u_0 and (b) transverse displacement w_0 at sensor location S on the mid-plane of the panel under 5-cycle tone burst excitation with an excitation frequency of 100 kHz. Numbers in bracket indicate total number of DOFs.	74
3.7	Configuration of panel considered for wave propagation analysis showing (a) pin-force model actuation at transducer edges B' and C' in the 2D model and (b) equivalent force system in 1D models. S denotes the sensing location. . . .	75
3.8	Axial displacement-time history response at location S on the top surface of the panel under 5-cycle tone burst signal actuation. Numbers in bracket indicate total number of DOFs.	77
3.9	Comparison of errors of TOT, FSDT and EBT solutions with respect to 2D elasticity solution for A_0 mode response of u for guided wave propagation . . .	78
3.10	In-plane displacement u at sensor location S on top surface of the panel under 5-cycle modulated tone burst excitation using under-integration for mass matrix	79
3.11	Hilbert envelope of wave response used for computing peak amplitudes of wave modes and their ToAs (shown for 50 kHz modulated tone burst excitation) . .	81
3.12	In-plane displacement u at sensor location S on top surface of the panel under 5-cycle modulated tone burst excitation using full integration for mass matrix .	83
3.13	Convergence of peak amplitude (PA) and ToA of S_0 mode obtained using under-integration of mass matrices: comparison with conventional FE	84
3.14	Convergence of peak amplitude (PA) and ToA of S_0 mode obtained using full integration of mass matrices: comparison with conventional FE	84

3.15	Convergence of peak amplitude (PA) and ToA of A_0 mode obtained using under-integration of mass matrices	85
3.16	Convergence of peak amplitude (PA) and ToA of A_0 mode obtained using full integration of mass matrices	86
3.17	In-plane displacement responses at S of the panel under 5-cycle tone burst excitation, obtained with different number of nodes per element, using under-integration approach. Figures in bracket for TOT SE are $n_e \times n$	87
3.18	In-plane displacement responses at S of the panel under 5-cycle tone burst excitation, obtained with different number of nodes per element, using full integration approach. Figures in bracket for TOT SE are $n_e \times n$	88
3.19	Convergence in the peak amplitude (PA) of A_0 mode with the number of nodes per element (n) for cases of under- and full integration	89
3.20	(a) Input impulse load on the beam. (b) Frequency spectrum of the impulse . .	89
3.21	Configuration of the beam under impact loading. Measurement is taken at the impact location S	90
3.22	Transverse velocity at the impact location near the pinned end of a beam, showing reflections from the boundary. *(Gopalakrishnan et al., 1992)	90
3.23	Natural frequencies of first 20 modes of the simply-supported composite beam with different lay-up: (a) $[0^\circ/90^\circ/90^\circ/0^\circ]$ and (b) $[0^\circ/90^\circ/0^\circ/90^\circ]$	92
3.24	Configuration of the composite panel $[0^\circ/90^\circ]_S$ considered for the wave propagation problem of Barouni and Saravanos (2016), showing force application in (a) 2D FE model and (b) 1D FE models	93
3.25	(a) Hann window modulated 5-cycle sinusoidal tone burst signal of 100 kHz central frequency. (b) Frequency spectrum of the signal	93
3.26	Transverse displacement profile of the laminated composite strip at $200 \mu s$ under a transverse modulated tone burst excitation of 100 kHz frequency. # (Rekatsinas et al., 2015), *(Barouni and Saravanos, 2016)	94

LIST OF FIGURES

3.27	Configuration of panel considered for wave propagation analysis showing (a) pin-force model actuation at transducer edges B' and C' in the 2D model and (b) equivalent force system in 1D models. S denotes the sensing location. . . .	95
3.28	In-plane displacement at location S on the top surface of the composite panel under 5-cycle tone burst signal actuation	97
3.29	In-plane displacement at S on the top surface of the composite panel, showing the difference between the CLT and 2D FE solutions for A_0 mode under 150 kHz excitation	98
3.30	Variation of error in 1D TOT, FSDT and CLT solutions with respect to 2D FE solution with excitation frequency for the A_0 mode response of u under 5-cycle tone burst excitation	98
3.31	Comparison of spectral and conventional FE solutions for in-plane displacement u at sensor location S on top surface of the composite panel under 5-cycle modulated tone burst excitation	100
3.32	Convergence of the SE and conventional FE solutions for the peak amplitude (PA) and time of arrival (ToA) of S_0 mode	102
3.33	Convergence of the SE and conventional FE solutions for the peak amplitude (PA) and time of arrival (ToA) of A_0 mode	102
3.34	Convergence in the peak amplitude (PA) of A_0 mode with the number of nodes per element n	103
3.35	(a) Input impact load at the tip of the cantilever beam. (b) Frequency spectrum of the loading	104
3.36	Transverse velocity at the tip of a cantilever composite beam $[0_5^\circ/90_5^\circ]$ under impact loading	105
4.1	Configuration of a laminated smart strip equipped with piezoelectric patches and containing multiple delaminations, showing structural axes x and z , and layer numbers starting from bottom.	111

4.2	(a) Regions containing intact laminates I_i and sub-laminates D_i separated by delaminated interfaces around a delamination zone. (b) Spectral element model of these regions having a separate set of elements for each sub-laminate with common nodes at delamination fronts.	112
4.3	(a) Geometry details and local reference plane of a sub-laminate and (b) Approximation of electric potential ϕ along thickness direction of the sub-laminate.	112
4.4	Spectral strip element based on the ZIGT, showing n physical nodes and an electric node: Distribution of physical nodes is unequal with GLL points spacing.	121
4.5	(a) A delamination front connecting an intact laminate (I) and a delaminated sub-laminate (D) having p number of layers starting from b^{th} interface of the intact laminate, and (b) Variations of u in laminates I and D at the delamination front. They should be ideally continuous at all points over sub-laminate thickness h_d	125
4.6	A patch front connecting the host laminate (H) and the patch transducer laminate (T). The variations of u in adjacent laminates H and T at a patch front are shown. They should be ideally continuous at all points over the host laminate thickness.	129
5.1	(a) Lay-up of the fiber metal laminate (FML) panel. The laminate comprises of interlacing layers of aluminium sheets and graphite-epoxy composite. (b) Natural frequencies of first 40 flexural modes for the simply-supported FML panel. Present SE results are validated with analytical ZIGT solution and compared with exact elasticity- and TOT-based solutions.	137
5.2	(a) Hann window modulated five-count pulse signal with $f = 100$ kHz and (b) its Fourier transform	139

5.3	Layout of four layer composite strip $[0^\circ/90^\circ]_S$ and force application for the Lamb wave propagation problem of Barouni and Saravanos (2016) in (a) the two-dimensional FE model and (b) the one-dimensional FE models. (c) Deflection of the composite panel $[0^\circ/90^\circ]_S$ at time $t = 200 \mu s$ under transverse Hann window modulated pulse with $f = 100$ kHz. The present solution shows a much better match with the reference LWT-based semi-analytical solution (Barouni and Saravanos, 2016)* than the HOT-based SE solution (Rekatsinas et al., 2015)#.	139
5.4	Layout of the strip for studying Lamb wave propagation excited by surface-pasted transducer: (a) 2D model with pin force model excitation at B' and C' , representing voltage excitation of a piezoelectric patch actuator bonded to the top surface of the strip at $B'C'$. (b) 1D model with the equivalent force system due to transfer from top surface to reference plane of the model. S is the response position.	141
5.5	Time history of in-plane displacement at point S on composite panel's top surface under surface shear actuation with five-count modulated pulse signal: Comparison of ZIGT and TOT solutions with 2D FE solution shows the effect of zigzag nature of through-thickness variation of u on the accuracy.	142
5.6	Time-history of inplane displacement at point S on the top of FML strip under surface shear actuation with five-count modulated pulse excitation: Comparison of different laminate theories with 2D FE solutions	144
5.7	Effect of excitation frequency on the accuracy of the ZIGT, TOT, and FSDT in predicting A_0 mode amplitude for u . The error is computed with respect to continuum FE solutions.	145
5.8	Comparison of present SE and standard FE solutions for the A_0 mode of u at point S on the top of FML strip under five-count modulated pulse excitations of different frequencies	147
5.9	A typical wave propagation response and its Hilbert envelop for identifying the peaks of waveform and their respective ToAs.	148
5.10	Convergence of the spectral and standard finite element solutions based on ZIGT for S_0 mode of the FML panel. The error is computed with respect to the highly converged standard FE solution based on the ZIGT.	149

5.11	Convergence of the spectral and standard finite element solutions based on ZIGT for A_0 mode of the FML panel. The error is computed with respect to the highly converged standard FE solution based on the ZIGT.	149
5.12	Configuration of eight-layer composite beam showing three cases of delaminations. The response is computed for two boundary conditions – cantilever and simply-supported.	152
5.13	Hann window modulated tone burst signal having seven cycles with $f = 200$ kHz showing its (a) time-history and (b) frequency spectrum (Fourier transform)	156
5.14	Schematic configuration of the composite panel $[0^\circ/90^\circ/0^\circ]$ containing a delamination (BC) of length 50 mm at the first interface from bottom for the problem of Ramadas et al. (2010), showing the transverse force application in (a) the 2D FE model, (b) the present 1D SE model, and (c) deflection response at the sensor location S due to excitation of seven-cycle Hann window modulated signal of central frequency $f = 200$ kHz at A.	156
5.15	Schematic configuration of the six-layer composite panel containing delamination for studying its effect on Lamb wave propagation induced by a PWAS patch pasted at $B'C'$. (a) Actuator force application in the 2D FE model according to the pin-force method, (b) application of equivalent force system in the 1D model, and (c) laminate lay-up showing the location of delamination interface. The response is recorded at S.	158
5.16	Portion of 2D FE model in ABAQUS for the delaminated six-layer composite panel showing the mesh: (a) undeformed and (b) deformed configurations. . . .	159
5.17	Comparison of ZIGT- and TOT-SE solutions with continuum-based 2D FE simulation in ABAQUS for $u(t)$ due to surface-bonded transducer actuation at $B'C'$ with a modulated five-cycle tone burst signal. The output is recorded at sensor location S on the top surface of the delaminated panel.	160
5.18	Comparison of solution accuracy of present SE and standard FE models of the same size for the A_0 mode and the delamination induced extra mode of the axial displacement $u(t)$ recorded at S on panel's top surface (Fig. 5.15) due to modulated five-cycle tone burst excitation at $B'C'$	162

5.19	Effect of interfacial location of delamination on the in-plane displacement $u(t)$ measured at sensor location S on the top surface of the panel due to Hann windowed five-count tone burst excitation with $f = 100$ kHz	163
5.20	Variation of transverse kinetic energy parameter E_w along the top surface of the panel between the actuator and sensor for different longitudinal locations of delamination introduced at (a) bottommost, (b) middle, and (c) topmost interfaces. The up-arrows on the x -axis indicate the positions of the peaks of the E_w profiles, which are seen to almost coincide with the centers of delaminations, clearly identifying their presence.	164
6.1	Configuration of hybrid laminated composite beam having (a) continuous piezoelectric layer and (b) piezoelectric patch bonded to the top surface of elastic laminate.	170
6.2	(a) Configuration of a cantilever aluminium beam featuring PZT-5H patches bonded at its top and bottom surfaces with a center distance d_P from the fixed end. (b) Variation of the tip deflection $w(l,0)$ due to applied electric potential of ± 10 V at the piezo-patch outer surfaces with the patch position d_P	174
6.3	Mode shapes of cantilever smart beam with $S = 10$ for first four natural frequencies: Comparison of the present method, with and without continuity at the patch fronts, to the piezoelasticity-based FE solution.	176
6.4	Mode shapes of cantilever smart beam with $S = 20$ for first four natural frequencies: Comparison of the present method, with and without continuity at the patch fronts, to the piezoelasticity-based FE solution.	177
6.5	Schematic configuration of the plate strip with piezoelectric patches bonded to the top surface of the host plate for Lamb wave propagation analysis. The response is obtained (a) with PZT sensor and (b) without PZT sensor.	177
6.6	Normalized sensor voltage due to Hann windowed sinusoidal pulse signal with $f = 200$ kHz: Comparison with the experimental data and piezoelasticity-based 3D FE solution. *(Agrahari and Kapuria, 2016b)	178

6.7	Transverse acceleration $\ddot{w}$ at the sensor location generated due to 2.5-count modulated tone burst excitation voltage at the top of the piezo-patch bonded on the composite cantilever beam at $f = 3$ kHz: (a) Comparison with the continuum-based FE solution and (b) comparison of current solutions, with and without accelerometer mass and property corrections, with the experimental data. ^a (Ip and Mai, 2003), ^b (Ip and Mai, 2004)	180
6.8	Experimental set-up for Lamb wave generation and sensing with patch PZT transducers in an aluminium plate.	181
6.9	Normalized sensor voltage obtained from experiment due to excitation on the top of the PZT actuator using five-cycle Hann windowed tone burst voltage signal with $f = 180$ kHz: Comparison of the present model with and without patch front continuity.	181
6.10	Sensor voltage signal $\phi(t)$ due to the excitation at the patch's top surface with a modulated five-cycle tone burst signal: Comparison of the ZIGT- and TOT-based SE solutions with piezoelectricity-based FE solution	184
6.11	Time-history of u at the sensor location S due to the excitation at the patch's top surface with a modulated five-cycle tone burst signal: Comparison of the ZIGT- and TOT-based SE solutions with piezoelectricity-based FE solution	185
6.12	Through-thickness distribution profile of u along the sensor location S utilizing the ZIGT- and TOT-based SE for (a) S_0 mode at $t = 52\mu s$ and (b) A_0 mode at $t = 135\mu s$: Comparison with piezoelectricity-based FE solution.	186
6.13	Comparison of the sensor voltage signal $\phi(t)$ using the present method with and without employing continuity model at the patch fronts for piezoelectric thickness h_P (a) 0.25 mm and (b) 0.5 mm. The excitation for this case is Hann-window modulated five-cycle tone burst signal with $f = 100$ kHz.	187
6.14	Configuration of the smart composite panel with a notch-type damage for Lamb wave propagation analysis. The response is recorded at the sensor location S at the panel's top surface.	187

6.15	Comparison of the current SE model with its counterpart standard FE of identical model size for in-plane displacement $u(t)$ at sensor location S due to modulated tone burst excitation signal ($f = 100$ kHz) applied at the actuator for (a) undamaged condition and (b) notch-type damage.	188
6.16	Comparison of the current SE model with its counterpart standard FE of identical model size for in-plane displacement $u(t)$ at sensor location S due to modulated tone burst excitation signal ($f = 250$ kHz) applied at the actuator for notch-type damage.	188
6.17	Configuration of a cantilever six-layer Kevlar composite panel having $[0^\circ]_6$ lay-up equipped with a PZT actuator bonded at panel's top surface. The laminate contains a delamination of 25 mm length at the second interface from the bottom. The responses are recorded at the sensor location S.	190
6.18	(a) Transverse acceleration $a (= \ddot{w}_0)$ and (b) transverse displacement w_0 at the sensor location 'S' due to the excitation voltage applied at the top of the PZT bonded on the panel's top surface with $f = 3$ kHz: Comparison with the continuum-based FE solution.	191
6.19	(a) Configuration of the smart composite panel with the PFRC actuator (A_1A_2) and sensor (S_1S_2) bonded to the panel's top surface for generation and sensing the Lamb wave signal to study the interaction of the generated signal with delamination (E_1E_2). (b) Laminate lay-up showing the delaminated interface.	192
6.20	Sensor voltage signal $\phi(t)$ due to the actuator excitation with a modulated five-cycle tone burst signal: Comparison of the ZIGT- and TOT-based SE solutions with piezoelectricity-based FE solution for a composite panel having delamination. The extra mode indicates the presence of delamination damage.	194
6.21	Sensor voltage signal $\phi(t)$ due to the excitation at the patch's top surface with a modulated five-cycle tone burst signal: Comparison of the ZIGT-based SE and conventional FE with the reference solution for a delaminated panel	195
6.22	Effect of delamination interface location on the sensor voltage $\phi(t)$ under Hann-windowed five-count tone burst signal actuation with $f = 100$ kHz.	196

- 6.23 Effect of delamination size (a_{del}) on the sensor voltage response $\phi(t)$ for a smart composite panel. Excitation is Hann-windowed five-count tone burst signal with $f = 100$ kHz. Delamination is at the first interface from bottom of the laminate. 198

List of Tables

2.1	Lobatto polynomials	31
2.2	Coordinates of Lobatto nodes (ξ_i) and corresponding weights (W_i) for Lobatto integration quadrature	33
2.3	Non-dimensionalised natural frequencies ($\bar{\omega}_n = \omega_n \frac{L^2}{h} \sqrt{\frac{\rho}{E}}$) of first 20 modes of a cantilever beam obtained using the direct explicit method	42
2.4	Non-dimensionalised natural frequencies ($\bar{\omega}_n = \omega_n \frac{L^2}{h} \sqrt{\frac{\rho}{E}}$) of first 20 modes of a cantilever beam obtained using the Guyan reduction method	43
2.5	Comparison of the accuracy and computational efficiency of the present spectral beam element using direct explicit method of solution with the conventional FEM for wave propagation analysis of undamaged beam	46
2.6	Comparison of the accuracy and computational efficiency of the present SE method using Guyan reduction method with the conventional FEM for wave propagation analysis of undamaged beam	49
2.7	Comparison of the accuracy and computational efficiency of the present SE method using Guyan reduction method with the conventional FEM for wave propagation analysis of beam with notch-type damage for 400 kHz excitation	52
3.1	Comparison of the accuracy and computational efficiency of the present TOT SE using under-integration with the conventional TOT FE for analysis of guided wave propagation in a panel	81

3.2	Comparison of the accuracy and computational efficiency of the present TOT SE using full integration with the conventional TOT FE for analysis of guided wave propagation in a panel	82
3.3	Nondimensional fundamental natural frequency $\bar{\omega}_1 = \omega_1 a^2 / h \sqrt{\rho / E_2}$ for laminated composite beams with different lay-ups and boundary conditions	91
3.4	Comparison of the accuracy and computational efficiency of the present TOT SE with the conventional TOT FE for analysis of guided wave propagation in a $[0^\circ/90^\circ]_S$ composite panel	101
4.1	Essential and natural boundary conditions	133
4.2	Boundary conditions for some commonly used supports	133
5.1	Material properties	137
5.2	Convergence and efficiency comparison between present SE and standard FE models based on ZIGT for analysing wave propagation problem of FML panel .	148
5.3	Material properties	151
5.4	Natural frequencies (Hz) of first ten flexural modes of a cantilever composite beam $([0^\circ/90^\circ]_{2S})$ having multiple delaminations of 50.8 mm length.	153
5.5	Natural frequencies (Hz) of first ten flexural modes of a cantilever composite beam $([0^\circ/90^\circ]_{2S})$ having multiple delaminations of 101.6 mm length.	154
5.6	Convergence study on mesh size of the SE model for natural frequencies of thick ($S=5$) composite beams $([0^\circ/90^\circ]_S)$ with delamination size of 50% of the span.	155
5.7	Natural frequencies (Hz) of thick ($S=5$) cantilever and simply-supported delaminated composite beams $([0^\circ/90^\circ]_S)$: Comparison of the two continuity models with 3D FE solution.	155
6.1	Material properties	171

6.2	Static and stress responses of laminate composite smart beams integrated with full piezoelectric layer, bonded on the top surface of the elastic laminate, under mechanical and potential loadings: Comparison with piezoelasticity-based FE solution	171
6.3	Static and stress responses of cantilever laminate composite smart beams integrated with piezoelectric patch, bonded on the top surface of the elastic laminate spanning from $x = 0$ to $0.3l$, under mechanical and potential loadings: Comparison with piezoelasticity-based FE solution	173
6.4	Natural frequencies (Hz) of first four modes of smart cantilever beam integrated with PZT patch: Comparison of the present method, with and without continuity model, to the reference piezoelasticity-based FE solution.	175

List of Abbreviations

CLT	= Classical lamination theory	TDSE	= Time-domain Spectral element
DEM	= Direct explicit method	ToA	= Time of arrival
EBT	= Euler-Bernoulli theory	TOT	= Third-order shear deformable theory
ELT	= Efficient layerwise theory	WFEM	= Wave finite element method
ESLT	= Equivalent single layer theory	ZIGT	= Zigzag theory
FDSEM	= Frequency-domain spectral element method		
FML	= Fibre-metal laminate		
FSDT	= First-order shear deformable theory		
GLL	= Gauss–Lobatto–Legendre		
GRM	= Guyan reduction method		
GW	= Guided wave		
HOT	= Higher-order theory		
HPLSC	= Hybrid pointleast squares continuity		
LWT	= Layerwise theory		
LS	= Least-square		
NDE	= Non-destructive evaluation		
PA	= Peak amplitude		
PC	= Point-continuity		
PFRF	= Piezoelectric fibre reinforced composite		
PZT	= Lead zirconate titanate		
PUFEM	= Partition of unity finite element method		
PWAS	= Piezoelectric wafer active sensors		
RMSD	= Root mean square difference		
SAFE	= Semi-analytical finite element		
SHM	= Structural health monitoring		
SSSD	= Signal sum of squares differences		