

**STOCHASTIC RESPONSE AND STABILITY ANALYSIS OF  
NONLINEAR SDOF SYSTEM IN PROBABILITY DOMAIN**

**By**

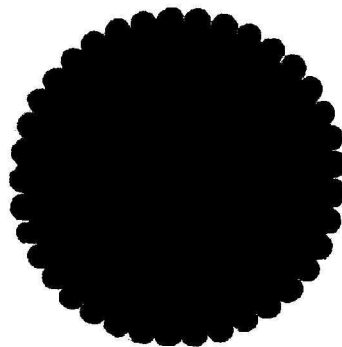
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**DEPARTMENT OF CIVIL ENGINEERING**

**Submitted**

**In fulfillment of the requirement of the degree of  
DOCTOR OF PHILOSOPHY**

**to the**



**INDIAN INSTITUTE OF TECHNOLOGY, DELHI**

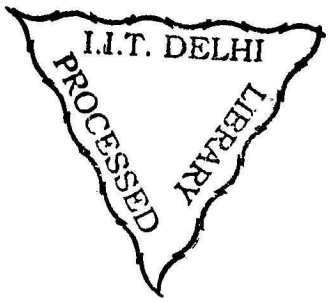
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**SEPTEMBER, 2008**

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## CERTIFICATE

This is to certify that the thesis entitled, “Stochastic Response and Stability Analysis of Nonlinear SDOF System in Probability Domain”, being submitted by Mr. Deepak Kumar, to the Indian Institute of Technology, Delhi, for the award of ‘DOCTOR OF PHILOSOPHY’ in Civil Engineering, is a record of the bonafide research work carried out by him under our supervision and guidance. He has fulfilled the requirements for submission of this thesis, which to the best of our knowledge has reached the requisite standard.

The material contained in the thesis has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.



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September, 2008

## **Acknowledgement**

I express my deep sense of gratitude to Prof. T. K. Datta and Prof. A. K. Jain, Department of Civil Engineering, Indian Institute of Technology, Delhi for their valuable guidance and constant encouragement during all phases of the present work. I am also indebted to Prof T. K. Datta for his help and encouragement during the work as well as his availability for the discussions. His great advice for my work as well as out of the work is always touching to heart.

I thank Dr. S. Karunes (Prof T. K. Datta's sister) for her lovely behavior as well as her offers for lovely lunch and dinner during my stay in IIT Delhi, which was not less than touch of a family.

I show a great respect to my parents for showing patience during my work which did not allowed me to meet them, some time for a long period. I respect them for bringing me up with a proper training, which allowed me to keep myself a better human and a better philosopher. I also show respect to my elder sister, Sanju Kumari and elder brother Sanjeev Kumar who were a part of bringing me up as well as motivating me continuously during my Ph. D. work. I also show respect to my brother-In-Law, Anil Kumar, for continuous motivation for the work, which he use to give me by different means.

I thank Dr. M. M. Rao, Mr. Amit Bundela and Mr. N. R. Gehlot for providing me the facilities and helping in computer lab.

Last but not the least, I thank my all friends (Samit Barai, Suvandan Saraswat, Bulu Pradhan, Anuj Sharma, A K Banik, Sandip Chaudhari, Umesh Pendharkar, B K Singh,

Diptesh Das, Pradeep Goyal and so on), who during my stay in IIT Delhi, accompany me for my entertainment and for providing me a circle which was not less than a family.

A handwritten signature in black ink, appearing to read 'Deepak Kumar', written in a cursive style. The signature is slanted upwards to the right.

DEEPAK KUMAR

## **Abstract**

For nonlinear single degree of freedom system subjected to both multiplicative and additive random excitations, use of stochastic averaging technique faces few difficulties. Firstly, the size of excitations should be small so that the response of the system converges weakly to a Markov process. Secondly, the excitations should preferably be broad banded so that the analytical results are reasonably accurate and finally, the nonlinear functions should be integrable if closed form expressions for the result are desired. As these issues are not widely reported in the literature, there is a need to examine them with a view to (i) show limiting values of the size parameter of excitations for which stochastic averaging technique can be applied to obtain reasonable estimates of PDF and mean square value of response; (ii) the effect of the nature of the frequency contents of excitation on the response and stability, and (iii) the use of the stochastic averaging method employing generalized harmonic functions for non-integrable functions representing the dynamic system. Further, there is a need to evolve a technique by which stochastic averaging procedure can be applied to practical problems having large size excitation. Although in recent years some numerical based techniques like intelligent Monte Carlo simulation with variance reduction technique are available for solving nonlinear stochastic differential equation without any limitation on size of excitation, the method requires large computational time for considering strongly nonlinear systems. Further, these methods cannot be effectively used for studying the stability of the system under different parametric variations. Close form solutions offer greater insight and advantage in studying the stability of the system. From these points of view stochastic averaging technique is useful at least for SDOF systems. Computational

time required is extremely small compares to the simulation analysis (as observed in the present study also). The other motivation for choosing stochastic averaging method for the present study is to show that there are many practical problems which can be adequately modeled as SDOF system and whose response and stability analyses can be elegantly performed using this technique. In fact application of the stochastic averaging technique for such kinds of problem form the challenge in research in this area. With the above requirements in view, the present work is under taken. Overall objectives of the work are (i) to study different limitations of the stochastic averaging method in obtaining the response and stability of SDOF system, (ii) to develop a method to correct the errors resulting from the limitations of the procedure and (iii) to carryout stochastic response and stability analyses of a class of flexible offshore structures namely, Articulated leg platform, Mooring lines and TLP tether under random wave loading.

In order to realize the objectives, a semi analytical method along with an error minimization technique is presented to perform response and stability analyses of nonlinear SDOF system in probability domain, which to a great extent, can overcome the limitations of the stochastic averaging procedure mentioned above. The semi analytical method consists of evolving a computational strategy to find averaged drift and diffusion coefficients in analytical forms using FFT and curve fitting technique for nonlinear functions, which cannot be integrated in closed form. The proposed error minimization technique is aimed at correcting the errors in drift and diffusion coefficients introduced due to sizable excitations. The procedure consists of finding a replacement system which is closest to the original system in some statistical sense and is obtained by classical optimization techniques. The proposed method is useful for determining the response of the nonlinear system whose nonlinearities cannot be handled

in closed form and whose sizes of excitation and damping are not small. Further, the proposed method enables one to study the stability of the system by examining the asymptotic stability of the two boundaries of the system even when the nonlinearities cannot be handled in closed form. A number of numerical studies with different types of nonlinearities and different types of excitations namely, broad to narrow band are conducted in order to investigate the limiting size of excitation for which stochastic averaging technique can be applied and illustrate the performance and efficiency of the proposed technique for different cases of study. As an application to real life problems, the proposed techniques are used to perform the response and stability analyses of two articulated tower platforms, a taut and a flexible mooring systems, all modeled as a SDOF system. Further, stochastic stability of transverse oscillation of a TLP tether is investigated. For the purpose of above study, three random sea states with 21m, 16m, and 8m significant wave heights are considered.

The work is presented in seven chapters. In the first two chapters, an introduction to the work done and pertinent review of literature is presented. Third chapter deals with the development of the semi analytical method and error minimization technique for finding the response and stability of the system subjected to large size excitation. Fourth chapter deals with detailed parametric study to bring out the limitations of the application of stochastic averaging technique to different cases. Also, the effectiveness of the proposed method for solving different types of nonlinear SDOF system subjected to large size additive and multiplicative loading for obtaining their responses and investigating their stability conditions is illustrated. Chapters five through seven show the application of the proposed

method for performing response and stability analyses of flexible offshore structures namely, Articulated Leg Platform, Taut and Flexible Mooring systems and TLP tether.

Numerical studies conducted in different chapters show that the proposed semi analytical technique along with the error minimization procedure can be used for practical problems which may be idealized as SDOF systems subjected to sizable additive and multiplicative excitations without any restriction on the form of nonlinearities.

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