# INVESTIGATION OF BUBBLE INTERACTION WITH GAS-LIQUID AND LIQUID-LIQUID INTERFACE

GHULAM RABBANI




DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY DELHI

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# INVESTIGATION OF BUBBLE INTERACTION WITH GAS-LIQUID AND LIQUID-LIQUID INTERFACE

by

**GHULAM RABBANI**

Department of MECHANICAL ENGINEERING

Submitted

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*I would like to dedicate this thesis to my loving parents*

# Certificate

This is to certify that the thesis entitled **"INVESTIGATION OF BUBBLE INTERACTION WITH GAS-LIQUID AND LIQUID-LIQUID INTERFACE"**, submitted by **GHULAM RABBANI** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in MECHANICAL ENGINEERING, is a record of the original, bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.

**Prof. BAHNI RAY**
Department of MECHANICAL ENGINEERING,
Indian Institute of Technology Delhi.

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# *Abstract*

Bubble dynamics plays a crucial role in a wide range of chemical process industries, where understanding bubble formation, behavior in different fluid environments, and interaction with interfaces is vital for optimizing reactor performance, enhancing mixing efficiency, and improving overall production rates. This dissertation systematically investigates the interaction of bubbles with gas-liquid and liquid-liquid interfaces and is presented in four comprehensive parts. In the first part, the study revisits the fundamental problem of bubble formation in liquids of extreme viscosities, addressing discrepancies in the literature. Bubbles were generated using needles of diameters ranging from 0.84 mm to 1.54 mm at gas flow rates between 5 ml/min and 80 ml/min. The experiments reveal distinct behaviors in low-viscosity fluids (e.g., deionized water), where bubble necking time remains nearly constant with gas flow rate, and daughter bubbles emerge due to secondary necking. In contrast, in high-viscosity fluids (e.g., glycerol), smaller bubbles were observed to slide over larger ones and coalesce symmetrically along the axis. The interaction point (whether at an angle or inline interaction) between the bubbles plays an important role in determining whether bubbles coalesce or slide past each other. Additionally, a new observation of bubble pairing (doublet formation) is reported at low gas flow rates with larger needle diameters. The second part focuses on the dynamics of bubble bursting at the gas-liquid interface, extending from bubble formation to satellite droplet ejection. The influence of surfactant concentration (ranging from below to above the critical micelle concentration, CMC) and non-dimensional liquid height (h/D) is explored. Experiments are conducted with a fixed needle diameter of 1.36 mm in different surfactant concentrations. The study shows that the bubble size is largest in clean water and decreases with surfactant concentration. In the presence of high surfactant concentrations, the aspect ratio of the bubble ranges from 1 to 1.5, and crater formation after bursting becomes less pronounced. Surfactants delay both the time required for a bubble to reach the interface and the bursting time of the bubble. At high concentrations, satellite droplet ejection is suppressed entirely. Interestingly, satellite droplet size and height exhibit monotonic trends with surfactant concentration and non-monotonic trends with h/D, whereas crater depth is influenced by bubble size, film thickness, and surface tension. The third part of the research focuses on bubble interactions with thin and thick liquid-liquid interfaces

with relevance to oil-spill mitigation. The viscosity of the upper (oil) layer is varied from 0.65 to 1000 cSt, while the lower phase (bulk) is water under surfactant-free and surfactant-laden conditions. For thin interfaces ( 1 mm), we focus on bubble bursting at the interface, including cavity collapse and the formation of a satellite droplet. The formation of oil droplets in the bulk is studied for both single-bubble and bubbly-flow interactions with the liquid-liquid interface, in the presence and absence of surfactant. After the system reaches a steady state, DLS measurements are performed on samples at 1 h and 24 h to examine the size and stability of the oil-in-water emulsion. The present study is also extended to a thick upper layer ( 4 cm), where all possible outcomes of bubble collisions with the liquid-liquid interface are observed, including bubble trapping at the interface, water jet piercing into the bubble after jet entrainment, and jet breakup into single or multiple droplets. An attempt has been made to summarize these outcomes using a regime map based on bond number and retention time. Overall, this dissertation provides new insights for understanding bubble-interface interactions in complex fluid systems, contributing to the development of more efficient and controlled processes in industrial applications involving multiphase flows.

# *Abstract*

बुलबुलों की गतिशीलता रासायनिक प्रक्रिया उद्योगों की अनेक प्रणालियों में अत्यंत महत्वपूर्ण भूमिका निभाती है। बुलबुले के निर्माण, विभिन्न द्रव माध्यमों में उसके व्यवहार तथा विभिन्न इंटरफेस के साथ उसकी पारस्परिक क्रिया को समझना अभिक्रियक के प्रदर्शन को अनुकूलित करने, मिश्रण दक्षता बढ़ाने तथा समग्र उत्पादन दर में सुधार के लिए आवश्यक है। प्रस्तुत शोधप्रबंध में गैस-द्रव तथा द्रव-द्रव इंटरफेस के साथ बुलबुलों की अंतःक्रिया का व्यवस्थित अध्ययन किया गया है, जिसे चार प्रमुख भागों में विभाजित किया गया है।

प्रथम भाग में अत्यधिक श्यानता वाले द्रवों में बुलबुले के निर्माण की मूलभूत समस्या का पुनः परीक्षण किया गया है, जिससे पूर्व अध्ययनों में पाई गई असंगतियों को स्पष्ट किया जा सके। विभिन्न व्यास वाली सुइयों तथा अलग-अलग गैस प्रवाह दरों पर बुलबुले उत्पन्न किए गए। कम श्यानता वाले द्रवों में यह पाया गया कि बुलबुले के संकुचन का समय प्रवाह दर से लगभग अप्रभावित रहता है और द्वितीयक संकुचन के कारण छोटे बुलबुले बनते हैं। इसके विपरीत, अधिक श्यानता वाले द्रवों में छोटे बुलबुले बड़े बुलबुलों की सतह पर फिसलते हुए अक्ष के साथ सममित रूप से मिलते हैं। यह भी पाया गया कि बुलबुलों के बीच टकराव का प्रकार यह निर्धारित करता है कि वे आपस में मिलेंगे या एक-दूसरे के पास से गुजर जाएंगे। कम प्रवाह दर पर बुलबुलों की जोड़ी बनने की एक नई घटना भी देखी गई।

दूसरे भाग में गैस-द्रव इंटरफेस पर बुलबुले के फटने की प्रक्रिया का अध्ययन किया गया है, जिसमें बुलबुले के निर्माण से लेकर सूक्ष्म बूंदों के उत्सर्जन तक की घटनाओं को शामिल किया गया है। सर्फैक्टेंट की सांद्रता तथा विमाहीन द्रव ऊँचाई के प्रभावों की जाँच की गई। परिणाम दर्शाते हैं कि स्वच्छ जल में बुलबुले का आकार अधिकतम होता है और सर्फैक्टेंट की सांद्रता बढ़ने पर यह घटता है। अधिक सांद्रता पर बुलबुले का आकार अधिक गोलाकार हो जाता है और फटने के बाद बनने वाली गुहा कम स्पष्ट होती है। सर्फैक्टेंट बुलबुले के इंटरफेस तक पहुँचने तथा फटने की प्रक्रिया को विलंबित करते हैं। उच्च सांद्रता पर सूक्ष्म बूंदों का उत्सर्जन पूर्णतः समाप्त हो जाता है। सूक्ष्म बूंदों का आकार और उनकी ऊँचाई सर्फैक्टेंट तथा द्रव ऊँचाई से प्रभावित होती है, जबकि गुहा की गहराई बुलबुले के आकार, फिल्म की मोटाई और सतही तनाव पर निर्भर करती है।

तीसरे भाग में पतली तथा मोटी द्रव-द्रव इंटरफेस के साथ बुलबुलों की अंतःक्रिया का अध्ययन किया गया है, जिसका संबंध तेल रिसाव नियंत्रण से है। ऊपरी परत की श्यानता को व्यापक सीमा में बदला गया, जबकि निचला चरण जल रखा गया, सर्फैक्टेंट की उपस्थिति तथा अनुपस्थिति दोनों स्थितियों में। पतली इंटरफेस के लिए बुलबुले के फटने, गुहा ध्वंस और सूक्ष्म बूंद बनने की प्रक्रियाओं का विश्लेषण किया गया। एकल बुलबुले तथा बहु-बुलबुला प्रवाह की स्थिति में तेल की बूंदों के निर्माण का भी अध्ययन किया गया। स्थिर अवस्था प्राप्त होने के बाद नमूनों का विश्लेषण कर इमल्शन के आकार तथा स्थिरता का आकलन किया गया। मोटी परत की स्थिति में बुलबुले के टकराव के कई संभावित परिणाम देखे गए, जैसे इंटरफेस पर बुलबुले का रुक जाना, द्रव जेट का बुलबुले को भेदना, तथा जेट का एक या अनेक बूंदों में टूटना। इन परिणामों को विमाहीन संख्याओं के आधार पर विभिन्न क्षेत्रों में वर्गीकृत करने का प्रयास किया गया है।

समग्र रूप से, यह शोध जटिल द्रव प्रणालियों में बुलबुला-इंटरफेस अंतःक्रिया को समझने के लिए नए दृष्टिकोण प्रदान करता है। ये निष्कर्ष बहु-चरणीय प्रवाह से संबंधित औद्योगिक प्रक्रियाओं को अधिक कुशल, विश्वसनीय तथा नियंत्रित बनाने में उपयोगी सिद्ध होंगे।

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# Abbreviations

| | |
|---|---|
| **CMC** | **C**ritical **M**icelle **C**oncentration |
| **DLS** | **D**ynamic **L**ight **S**cattering |
| **CTAB** | **C**etyltrimethylammonium **b**romide |
| **DI** | **D**e **I**onized **W**ater **VOF** |
| **V**olume **O**f **F**luid | |
| **PFV** | **P**hotron **F**astcam**V**iewer |
| **CDH** | **C**entral **D**rug **H**ouse |
| **fps** | **F**rames **P**er **S**econd |
| **AR** | **A**spect **R**atio |
| **Ma** | **M**arangoni **N**umber |
| **W** | **W**ater |
| **SF** | **S**ilicon **F**luid or **O**il |
| **Re** | **R**eynold **N**umber |
| **Ca** | **C**apillary **N**umber |
| **Ar** | **A**rchimedes **N**umber |
| **Bo** | **B**ond **N**umber |
| **We** | **W**eber **N**umber |
| **Oh** | **O**hnesorge **N**umber |

# Symbols

| | |
|---|---|
| $D$ | needle diameter |
| $d$ | bubble diameter |
| $h$ | distance between needle exit and interface |
| $h_f$ | liquid film thickness |
| $\rho$ | fluid density |
| $\mu$ | fluid viscosity |
| $V_b$ | bubble volume |
| $c$ | bulge radius |
| $C$ | concentrations |
| $\sigma$ | surface tension |
| $\sigma_I$ | interfacial tension |
| $Z_c$ | crater depth |
| $Q$ | flow rate |
| $\Gamma$ | surfactant adsorption at particular time instant t |
| $\Gamma_\infty$ | maximum surfactant adsorption |
| $A$ | needle area |