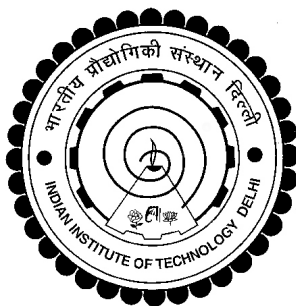


LAGRANGIAN TRACKING, ANALYSIS AND MODELING OF VELOCITY-GRADIENT DYNAMICS IN COMPRESSIBLE TURBULENCE

MD. DANISH



DEPARTMENT OF APPLIED MECHANICS
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by

MD. DANISH

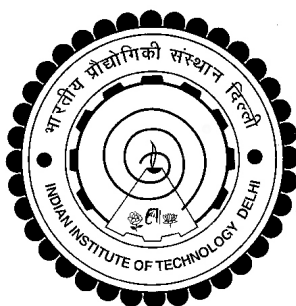
Department of Applied Mechanics

Submitted

in fulfillment of the requirements for the degree of

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CERTIFICATE

This is to certify that the thesis entitled “**Lagrangian Tracking, Analysis and Modeling of Velocity Gradient Dynamics in Compressible Turbulence**”, being submitted by **Md. Danish** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy**, is a record of the bonafide research carried out by him which has been prepared under our supervisions in conformity with the rules and regulations of the Indian Institute of Technology Delhi. The research reports and results presented in the thesis has not been submitted for any degree or diploma in any other university or institute.

Dr. Sawan S. Sinha

Assistant Professor

Department of Applied Mechanics

Indian Institute of Technology Delhi

New Delhi - 110016

India

Dr. Balaji Srinivasan

Associate Professor

Department of Applied Mechanics

Indian Institute of Technology Delhi

New Delhi - 110016

India

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ABSTRACT

Velocity gradient tensor and its dynamics is central to many important turbulence processes like mixing, energy-cascade, material element deformation, intermittency etc. The study of velocity gradient tensor is not only essential from the point of view of improving our understanding of turbulence, but also for the development of better turbulence closure models. In compressible turbulence the velocity gradient tensor is expected to be different than incompressible turbulence. That difference arises because of the change in nature of pressure (thermodynamic), divergence of velocity (non-zero) and steep gradients in flow variables (shocklets). Thus, it is important to study the influence of compressibility on velocity gradients and in turn the influence of compressibility on processes like cascading, mixing etc. With this motivation, three specific studies are taken up in this thesis:

- (1) Study A: Role of compressibility on velocity-gradient–scalar-gradient interactions (scalar mixing),
- (2) Study B: Role of compressibility on pressure Hessian tensor, and
- (3) Study C: Role of compressibility on Lagrangian interactions of vorticity and strain-rate (energy cascade).

In study A, we investigate the role of compressibility on scalar-mixing *via* the scalar-gradient–velocity-gradient interactions. These interactions, in a statistically homogeneous scalar field of an isotropic compressible turbulent flow, represent three distinct mechanisms of scalar-mixing: (i) anisotropic-nonlinear, (ii) isotropic and (iii) viscous mechanisms. With an appropriately normalized form of these three mechanisms, we show that the isotropic mechanism depends only on the normalized dilatation, whereas anisotropic-nonlinear and viscous mechanisms seem to be dependent on both topology as well as normalized dilatation. The behavior of these two mechanisms (anisotropic-nonlinear and viscous) are further analyzed

through their DNS (direct numerical simulation) results jointly conditioned on normalized dilatation and topology. The joint-conditioned results of anisotropic-nonlinear mechanism indicate its dependency more on flow topology than on normalized dilatation, whereas the viscous mechanism seems to be fairly independent of topology and normalized dilatation. We further show that the joint-conditioned results of anisotropic-nonlinear mechanism exhibit almost a ‘universal’ behavior, independent of Mach number and Reynolds number. Subsequently, based on this study, we develop a Lagrangian model of anisotropic-nonlinear mechanism in terms of velocity-gradient tensor itself.

In study B, we investigate and understand the influence of compressibility on pressure Hessian tensor in compressible turbulence. For this purpose, we employ DNS dataset of compressible decaying isotropic turbulence. Based on the DNS-based study, we identify two important mechanisms that significantly influence the evolution of pressure Hessian tensor in compressible turbulence. These mechanisms are: (i) non-symmetric, non-isentropic mechanism and (ii) symmetric mechanism. The physics of these two mechanisms are, subsequently, incorporated to improve the existing Lagrangian model of pressure Hessian evolution equation (enhanced homogenized Euler equation model (EHEE)). With these modifications to the EHEE model, the improved model (new model) has been extensively evaluated over a range of Mach and Reynolds numbers. The results of new model show substantial improvements over the existing EHEE model.

In study C, we perform a Lagrangian study by tracking large number of fluid particles in almost time-continuous DNS datasets. The study primarily focus on the Lagrangian statistics of vorticity–strain-rate interactions *via* the alignment between vorticity and strain-rate eigenvectors. The influence of compressibility on the alignment statistics are investigated in terms of turbulent Mach number, topology and normalized dilatation. To contrast the effects of compressibility, if any, we also present corresponding results from incompressible decaying isotropic turbulence. We find that, like in incompressible turbulence, the vorticity shows

a distinct tendency to align with the initial direction of the largest strain-rate eigenvector; however the extent of this alignment tendency seems to be weaker in compressible flow than in incompressible flow. Based on the alignment statistics jointly conditioned on topology and normalized dilatation, we find that the weakened tendency of the vorticity vector to align with the largest eigenvector in compressible flows is attributable to the fluid elements having initially negligible dilatation and associated with stable-focus-stretching flow topology.

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