

OPTIMAL TRADING STRATEGIES UNDER DIFFERENT EXECUTION PRICE MODELS

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OPTIMAL TRADING STRATEGIES UNDER DIFFERENT EXECUTION PRICE MODELS

by

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Dedicated to my family

Certificate

This is to certify that the thesis entitled “**Optimal trading strategies under different execution price models**” submitted by **Ms. Arti Singh** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bona fide research work carried out by her under my supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

The optimal trading problem is a problem of fundamental importance among market practitioners, algorithmic traders, academicians, etc. It is not efficient to trade all the required number of shares of an asset at once, as doing so moves asset's price considerably if the trade size is large. Execution price is the price at which actual trading takes place, and which includes price impact of trade order and other factors. Investment performance is substantially related to the execution cost, which in turn depends upon the underlying execution price model. Thus it is important to efficiently model the execution price dynamics by including an appropriate dynamics for no-impact price path and proxies for other factors. However, at the same time it is important too that the optimal trading problem formulation under some execution price model should fall in the class of optimization problems which have nice properties associated with it.

In this thesis many possible models of execution price path for single asset and portfolio trading are proposed. The proposed price models are based on many theoretical and empirical evidences such as presence of higher order autoregressive behavior in financial time series of prices, presence of cointegration behavior in many groups of financial time series of prices, etc. Further, under some conditions on execution price models the corresponding optimal trading problems are proved to be convex programming problems. As it is well known that the class of convex programming problems exhibits many nice properties, like here any local minimum is a global minimum, the proposed execution price models do not add any computational complexity in the optimal trading problem.

By splitting a trading order into smaller packages to be traded periodically, a trader is exposed to long trading duration. This results in timing risk which is the risk occurs due to unfavorable price movements of the assets in long trading duration. Thus it is plausible to include a measure for the timing risk in the optimal trading problem formulation. Under different measures for timing risk, many mean-risk optimal trading problems are studied in literature. In this thesis the risk measures like variance, conditional value-at-risk are considered. With these risk measures the mean-risk optimal trading problems under the proposed execution price models are formulated. These problems are further proved to belong to class of convex programming problem under certain conditions on execution path. Apart from mean-risk models, another direction of optimal trading under risk which is considered in this thesis, is based on the use of stochastic dominance. The importance of stochastic dominance, specially second order stochastic dominance (SSD), is well recognized in the area of port-

folio optimization problem. However, in the area of optimal trading the use of SSD criteria for modeling timing risk measure is limited. In view of this, in this thesis optimal portfolio trading under SSD criteria is studied.

By stochastic dynamic programming problem the closed form solutions of unconstrained optimal trading problem of minimizing expected execution cost under the proposed price model are derived. Although dynamic programming method allows to obtain closed form solution of unconstrained optimal trading problem, at the same time there are some computational difficulty associated with this method to solve these problems under some constraints. Thus we resort to use static approximation method, suggested in literature, to present the solution of our proposed optimal trading problems. Extensive numerical illustrations on simulated and the real market financial data, are provided which render the practical significance of the proposed execution price models and the optimal trading problems.

सार

इष्टतम व्यापारिक समस्या बाजार व्यापारियों, एल्गोरिथम व्यापारियों, शिक्षाविदों आदि के बीच मौलिक महत्व की समस्या है। सभी आवश्यक शेयरों का एक बार में व्यापार करना कुशल नहीं होता है, क्योंकि ऐसा करने से परिसंपत्ति की कीमत काफी बढ़ जाती है, यदि व्यापार आकार बड़ा है। निष्पादन मूल्य वह कीमत है जिस पर वास्तविक व्यापार होता है, और जिसमें व्यापार आदेश और अन्य कारकों का मूल्य प्रभाव भी शामिल होता है। निवेश प्रदर्शन निष्पादन लागत से काफी हद तक संबंधित है, जो की अंतर्निहित निष्पादन मूल्य मॉडल पर निर्भर करता है। इसलिए यह महत्वपूर्ण है कि नो-इंपैक्ट प्राइस पथ के लिए उचित गतिशीलता और अन्य कारकों के लिए प्रॉक्सीज को शामिल करते हुए निष्पादन कीमत गतिशीलता को कुशलता से मॉडल किया जाए। इसके अलावा यह भी महत्वपूर्ण है कि कुछ निष्पादन मूल्य मॉडल के तहत इष्टतम व्यापारिक समस्या अनुकूलन समस्याओं के वर्ग में होना चाहिए, जिनके अच्छे गुण हैं।

इस थीसिस में एकल संपत्ति और पोर्टफोलियो व्यापार के लिए निष्पादन मूल्य पथ के कई संभव मॉडल प्रस्तावित किए गए हैं। प्रस्तावित मूल्य पथ मॉडल कई सैद्धांतिक और व्यावहारिक प्रमाणों पर आधारित हैं जैसे कि कीमतों की वित्तीय समय श्रृंखला में उच्च आदेश की स्वसमाश्रयी व्यवहार की उपस्थिति, कीमतों की वित्तीय समय श्रृंखला के कई समूहों में संयोग व्यवहार की उपस्थिति, आदि। इसके अलावा, निष्पादन मूल्य मॉडल पर कुछ शर्तों के तहत, इसी तरह की इष्टतम व्यापारिक समस्याएं उत्तल प्रोग्रामिंग समस्याएं साबित की गई हैं। जैसा कि यह अच्छी तरह से ज्ञात है कि उत्तल प्रोग्रामिंग समस्याओं का वर्ग बहुत अच्छे गुण प्रदर्शित करता है, जैसे यहां कोई स्थानीय न्यूनतम वैश्विक स्तर है, प्रस्तावित निष्पादन मूल्य मॉडल इष्टतम व्यापारिक समस्या में कम्प्यूटेशनल जटिलता नहीं करते हैं।

एक व्यापारिक आदेश को आवधिक व्यापार करने के लिए छोटे पैकेजों में विभाजित करके, एक व्यापारी लंबी अवधि के व्यापार के लिए बाध्य होता है। इसके कारण टाइमिंग रिस्क हो जाता है, जो की लंबी अवधि में परिसंपत्तियों के प्रतिकूल मूल्य आंदोलनों के कारण होता है। इसलिये यह महत्वपूर्ण है कि इष्टतम व्यापारिक समस्या के निर्धारण में टाइमिंग रिस्क के लिए एक माप शामिल करें। टाइमिंग रिस्क के लिए विभिन्न मापों के तहत, कई मीन-रिस्क इष्टतम व्यापारिक समस्याओं को साहित्य में पढ़ा गया है। इस थीसिस में टाइमिंग रिस्क मापों जैसे कि वैरिएंस और कंडिशनल वैल्यू-एट-रिस्क का अध्ययन किया गया है। इन मापों के साथ प्रस्तावित निष्पादन मूल्य मॉडल के तहत मीन-रिस्क इष्टतम व्यापारिक समस्याएं निरूपित की गई हैं। इन समस्याओं को निष्पादन पथ पर कुछ शर्तों के तहत उत्तल प्रोग्रामिंग समस्याएं साबित किया गया है। मीन-रिस्क मॉडलस के अलावा, टाइमिंग रिस्क के तहत इष्टतम व्यापार की एक और दिशा जिसे इस थीसिस में अध्ययन किया गया है, स्टोकेस्टिक डॉमिनेंस के उपयोग पर आधारित है। स्टोकेस्टिक डॉमिनेंस का महत्व, विशेष रूप से सेकंड ऑर्डर स्टोकेस्टिक डॉमिनेंस (एस एस डी), पोर्टफोलियो अनुकूलन समस्या के क्षेत्र में अच्छी तरह से मान्यता प्राप्त है। हालांकि, इष्टतम व्यापार के क्षेत्र में टाइमिंग रिस्क के माप के मॉडलिंग के लिए एस एस डी मानदंड का उपयोग सीमित है। इसे देखते हुए, इस थीसिस में एस एस डी मापदंड के तहत इष्टतम पोर्टफोलियो व्यापार का अध्ययन किया गया है।

स्टोकेस्टिक डायनामिक प्रोग्रामिंग विधि द्वारा प्रस्तावित मूल्य मॉडल के तहत अपेक्षाकृत निष्पादन लागत को कम करने के स्वैच्छिक इष्टतम व्यापारिक समस्या का संवृत रूप समाधान निकाला गया है। यद्यपि डायनामिक प्रोग्रामिंग विधि स्वैच्छिक इष्टतम व्यापारिक समस्या का संवृत रूप समाधान प्राप्त करने की अनुमति देता है, साथ ही कुछ कन्सर्टेड्स के तहत इन समस्याओं को हल करने के लिए इस विधि के उपयोग में कुछ कम्प्यूटेशनल

कठिनाईयां होती हैं। इसलिये हम हमारे प्रस्तावित इष्टतम व्यापारिक समस्याओं के समाधान प्रस्तुत करने के लिए साहित्य में सुझाए गए स्टैटिक एपॉक्सिमेशन विधि का उपयोग करते हैं। सिम्युलेटेड और वास्तविक मार्केट फाइनेंशियल डेटा पर व्यापक संख्यात्मक चित्रण प्रदान किए गए हैं, जो प्रस्तावित निष्पादन मूल्य मॉडल और इष्टतम व्यापारिक समस्याओं के व्यावहारिक महत्व को प्रस्तुत करते हैं।

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List of Notations

Symbol	Meaning
$E(.)$	Expected function
z^+	$\max\{z, 0\}$
$\mathbb{R}$	Set of real numbers
$\mathbb{Z}$	Set of integers
$\mathbb{C}$	Set of complex numbers
$\text{Real}(z)$	Real part of a complex number z
$\mathbb{D}$	Unit disc in the complex plane, that is, $\{z \in \mathbb{C}; z < 1\}$
H_+	Right half plane of cartesian plane, that is, $\{z \in \mathbb{C}; \text{Re}(z) \geq 0\}$