

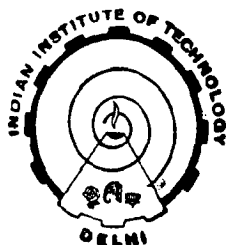
# **SPLINE APPROXIMATE METHODS FOR CERTAIN SECOND ORDER TWO-POINT BOUNDARY VALUE PROBLEMS**

by

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*A THESIS SUBMITTED  
IN FULFILMENT OF THE REQUIREMENTS  
FOR THE AWARD OF THE DEGREE OF*

**DOCTOR OF PHILOSOPHY**



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INDIAN INSTITUTE OF TECHNOLOGY  
HAUZ KHAS, NEW DELHI-110016  
1988**

CERTIFICATE

This is to certify that the thesis entitled "Spline Approximate Methods for Certain Second Order Two-point Boundary Value Problems" which is being submitted by Mr. R. Subramanian for the award of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology , New Delhi is a record of bonafide research work carried out by him under my guidance and supervision.

The thesis has reached the standard, fulfilling the requirements of the regulation relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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## ACKNOWLEDGEMENTS

I take this opportunity to record my profound sense of gratitude to Professor M.M.Chawla, FIMA under whose supervisor and able guidance I had the privilege to carry out my research work.

I am thankful to Professor M.K.Jain for his ever-readiness to help me. It is my pleasure to thank all my teachers who have always shown their keen interests in my work.

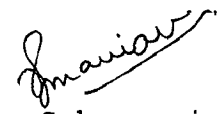
My thanks are also due to the authorities of the IIT Delhi for providing me with research facilities and financial support during the course of my research. I also thank Prof. R.K.Arora, Head, Computer Service Centre, for providing computer facilities.

I am greatly indebted to each and every member of my family, in particular, my father.

I wish to express my thanks to Mr. G.Chandrasekaran whom I shall remember for a long.

I also wish my sincere thanks to all my friends in particular Dr. S.Balasundaram, Dr. S.Gopalsamy, Dr. P.S.Rao and Messers S.Selvakumar, C.Thangaraj, A.Thayumanavan, Vikas Bist and Vijay Kumar who made my campus life a memorable one.

Finally, I thank Mr. Dev Raj Joshi for his excellent and accurate typing.

  
( R. Subramanian )

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