ROBUST OUTPUT FEEDBACK CONTROLLERS FOR NONLINEAR SYSTEMS

by

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CERTIFICATE

This is to certify that the thesis entitled "ROBUST OUTPUT FEEDBACK CONTROLLERS FOR NONLINEAR SYSTEMS" being submitted by AYYAGARI RAMA KALYAN to the department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of the bonafide research work carried out by him under my supervision and guidance. The results contained in this thesis have not been submitted to any other university of institute for the award of any degree or diploma.

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a. Ram...

RAMA KALYAN

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Abstract:

In this thesis, we address the robust control problem for a broad class of partially observed nonlinear systems. The major factors that hinder the design of a practical controller are (i) operation solely with observable quantities, (ii) implementation in finite time, and (iii) computational complexity.

The proposed work comprises three parts: (a) We formulate the H_∞ output feedback problem as a dynamic game and present analytical solutions in terms of an information state controller. We then introduce quadratic concave approximations to the infinite dimensional information state and address implementation issues. These approximations are designed to decrease the online computational complexity of the controller. We also address the numerical procedures for solving the Hamilton Jacobi inequalities and dynamic programming equations which arise in the solution to the H_∞ output feedback problem. (b) We provide a rigorous formulation of what may be called as generalized stochastic shortest path games. We establish the existence of an equilibrium value function for these games. We also show that the equilibrium cost-to-go function is characterized as the unique solution to a game-theoretic generalization

of Bellman's equation. We prove the convergence of the game-theoretic versions of dynamic programming algorithms. (c) We conclude the thesis by presenting a game-theoretic approach to reinforcement learning paradigm and establish the integration of the measures of robustness and intelligence of controllers.

We provide examples of continuous-time and discrete-time systems to validate our proposed analytical results.

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