

**SINGLE AND DOUBLE STAGE  
SHRUNKEN ESTIMATORS**

BY  
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A THESIS SUBMITTED TO THE  
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FOR THE AWARD OF THE DEGREE OF  
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DEPARTMENT OF MATHEMATICS  
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NOVEMBER, 1988

*TC*

*MY WIFE*

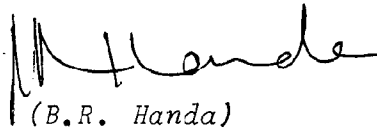
*and my sons*

*ALI AND HASSAN*

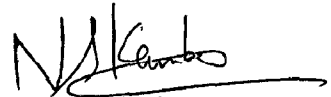
CERTIFICATE

This is to certify that the thesis entitled: "SINGLE AND DOUBLE STAGE SHRUNKEN ESTIMATORS" which is being submitted by Mr. Zuhair A. Hamid Al-Hemyari, Research Scholar, Mathematics Department to the Indian Institute of Technology, Delhi, for the award of the DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS, is a record of bonafide research work in STATISTICS carried out by him under our guidance and supervision and has fulfilled all the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



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