

MICROMECHANICAL ANALYSIS OF DEFORMATION AND FAILURE OF SNOW

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MICROMECHANICAL ANALYSIS OF DEFORMATION AND FAILURE OF SNOW

by

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DEDICATION

*I would like to dedicate my thesis to
My Parents and My Family*

CERTIFICATE

This is to certify that the thesis entitled “**MICROMECHANICAL ANALYSIS OF DEFORMATION AND FAILURE OF SNOW**” being submitted by **Mr. Chaman Chandel** is the report of bonafide research work carried by him under my supervision. This thesis has been prepared in conformity with the rules and regulations of the **INDIAN INSTITUTE OF TECHNOLOGY DELHI**. I further certify that the thesis has attained a standard required for the award of **Doctor of Philosophy** degree of the institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

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(Chaman Chandel)

ABSTRACT

A snowpack on a mountain slope is a layered structure with layers of strong and weak types of snow grains. The layers when loaded externally are acted upon by mixed loading (combined compressive and shear loading) which may cause failure of weak layer leading to avalanche initiation.

The strength of weak layers under combined compressive and shear loading are determined experimentally using a digital force gauge attached to a shear frame. Experiments were carried out on round grain snow layer (RGsr) and weak layers made of faceted snow (FCso) and near surface faceted particles (FCsf). Multi-axial stress response for FCsf snow layer was different from layers of RGsr and FCso snow. Positive correlation between shear strength and the normal compressive stress is observed for RGsr and FCso snow. The failure envelope in shear-compression stress space for FCsf snow layer is of elliptical shape.

Even though the constituting material of snow is ice, still response of different snow types is different. The modulus and strength of layers depends on the crystals which constitute the layer and are therefore micro-structure dependent. This dependence is studied by X-ray scanning the 3D micro-structure of different types of snow, constructing a representative volume element (RVE) and performing FE analysis on it under different loadings to obtain modulus and strength. For FE analysis, elasto-plastic properties of ice were assigned to the micro-structure. Numerical simulations were carried out on sub-volumes (V) equal to or larger than representative volume element ($V_{\phi_i}^{RVE}$). For RGsr snow the numerical simulations were validated by conducting an unconfined constant strain rate experiment on specially designed mechanical testing stage (MTS) of μ -CT scanner.

After establishing that the numerical simulations can represent actual behaviour of snow, the same was utilized to determine the ultimate strength and failure envelopes for FCso and FCsf snow type under combined compressive and shear loads. From the simulations it was concluded that the failure envelopes for FCso weak snow layer can be modeled with Mohr-Coulomb Cap model. The experimental measurements for FCso snow were made upto a load where only Mohr-Coulomb model was sufficient to model failure. The numerical simulations indicated that if sufficiently high normal compressive stress is applied then closed cap in addition to the MC model will also be observed. The simulated failure envelope for FCsf snow was an ellipse showed similar trend as observed from the experiments.

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NOMENCLATURE

G_f	The fracture energy release rate
S_T and S_u	The boundary
$\langle \rangle$	Averaging operator
$\bar{\sigma}$	Effective stress
h	Hardening modulus
β_o	Separated ice parts
β_1	The connectivity
σ'_{ij}	The deviatoric components of Cauchy stress
$\bar{\varepsilon}^p$	The equivalent plastic strain
ϕ_i	The ice volume fraction
β_2	The number of isolated pores fully surrounded by ice
D_E^2	The point variance of elastic modulus
$D_S^2(V)$	The variance of ultimate strength
ε_{abs}	Absolute error
ϖ	Damage variable
ρ_{th}	Density of thin weak layer
$\pm E_{\sigma_o}$	Error in X axis Intercept (compressive stress)
$\pm E_{\tau_o}$	Error in Y axis Intercept (Shear stress)
χ	Euler number
$\bar{\varepsilon}^p$	Plastic strain
$^{\circ}\text{C}$	Degree Centigrade
STD	Structural thickness density
$\bar{T}_i$	Prescribed external force per unit area on the boundary
ε_{rel}	Relative error
$V_{\phi_i}^{RVE}$	Representative volume element with respect to volume fraction

β_{1V}	The connectivity density
$D_Z^2(V)$	The fluctuations or variance
δ	The plastic displacement (beyond yield stress of ice)
δ_{pl}	The plastic displacement corresponding to the failure of ice
ε_f^{pl}	The Plastic strain at which final failure of material occurs
ε_0^{pl}	The plastic strain corresponding σ_{yo}
D_S^2	The point variance of ultimate strength
$\bar{u}_i$	The prescribed displacement on the boundary
σ_o	The yield stress
σ_{yo}	Yield stress at which damage initiates
σ	The compressive stress
Ω	Domain of macroscopic body
τ	Shear stress
σ_o	Intercept of X-axis (compressive stress)
τ_o	Y axis Intercept (shear stress)
ρ_s	Density of snow
ρ_u and ρ_l	Densities of homogeneous superstratum and substratum layer
μ	The absorption coefficient
μA	Micro ampere
μ -CT	X-ray micro-computed tomography
μm	micro meter
2D	Two Dimensional
3D	Three Dimensional
A	The snow density sampler's cross-sectional area
A_E	The integral range for Elastic modulus
A_s	The integral range for Ultimate strength
A_z	Integral range
C_{ijkl}	The stiffness coefficients
COV	Coefficient of variation

CT	Computer Tomography
E	Elastic modulus
E	Young's modulus
EHM	Equivalent homogeneous material
e_{kl}	strain tensor
σ_{ij}	Stress tensor
FCsf	Near surface faceted particles
V_e	Volume
FCso	Faceted snow
ϕ	Internal Friction
c	cohesion
FEM	Finite Element Method
f_i	Body force per unit volume
ϕ_{z_1}	The volume fraction of phase 1
G, G_i , G_o	The shear modulus
H	The snow density sampler height
h_t	Thickness of thin weak layer
kV	kilovolt
L	The characteristic length of each element
LW	Long Wave
m	Mass of sandwiched snow sample
M	Mean
ms	milli second
MTS	Mechanical testing stage
MCD	Minimum cut density
N	Number of independent (different) realizations
n_j	Outward unit normal vector on the boundary
$^\circ$	degree (angle)
RGsr	Round grain snow
RVE	Representative Volume Element

S	The ultimate strength
SSA	Specific surface area
D, D_c, D_s	Damage in snow
σ_c	Compressive Strength
τ_{max}	Shear strength corresponding to the common point of MC and its Cap
MCC	Mohr-Coulomb cap model
SW	Short Wave
u_i	The displacement vector
U_X	Displacement in x direction
U_Y	Displacement in y direction
U_Z	Displacement in z direction
V	Volume
x	the coordinate system of the macroscopic body
y	the coordinate system of the microscopic body
Z	Any effective property
α_E	Exponent for elastic modulus
α_s	Exponent for ultimate strength
σ_{ij}	The stress tensor
ν	Poisson ratio
$V_{\phi_i}^{RVE}$	RVE for ice volume fraction
V_E^{RVE}	RVE for modulus
V_{σ}^{RVE}	RVE for strength
n_3	Number of voxels
n_2	Number of voxel faces
n_1	Number of voxel edges
n_0	Number of voxel corners
$\frac{\partial T}{\partial x}$	Temperature gradient