THEORETICAL STUDIES ON BLOOD FLOW IN LARGE ARTERIES

by

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... Dedicated to

My Parents

for the uncompromising principles that guide their life ...

CERTIFICATE

This is to certify that the thesis entitled "THEORETICAL STUDIES ON BLOOD FLOW IN LARGE ARTERIES" being submitted by RANJANA MEHROTRA for the award of the degree of DOCTOR OF PHILOSOPHY, is a record of the original bonafide research work carried out by her. She has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis. The results presented in this thesis have not been submitted in part or full to any other University or Institute for award of any degree/diploma.

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PREFACE

The application of the mathematical models of cardiovascular flows can provide the clinician, medical or biological research worker with a great understanding of the complex phenomena associated with blood flows. Similarity of problems in Hydrodynamics and Cardiovascular Physiology has been responsible for the extensive exchange of ideas between the two disciplines. The importance of the physical aspects of blood flow in the circulation has contributed much to the theoretical developments in Hydrodynamics. Similarly, much of the characteristics of the circulation in both normal and pathological situations can be explained through a knowledge of Fluid Dynamics.

The object of the present thesis, which consists of six chapters, is to understand certain aspects of blood flows in the presence of stenosis - reduction in the lumen of the artery due to the gradual thickening of the arterial wall. The first chapter is an introductory one, which gives a brief description of the human cardiovascular system and discusses the pathological situation of atherosclerosis leading to a stenosis in the artery. It also includes a brief literature survey of the earlier theoretical models of cardiovascular flows which take account of the presence of stenosis. Throughout the thesis, blood is treated as a homogeneous incompressible fluid.

Chapter II, which consists of two parts, deals with the flow of an incompressible viscous fluid in a rigid elliptical tube of varying cross-section. The flow is considered laminar and fully developed. The geometry of the stenosis is introduced by specifying the change in area of cross-section of stenosed artery with axial distance. The first part deals with the steady flow in a stenosed artery while the second part takes account of the unsteady character of the flow. The distributions of velocity, pressure, shearing stress and impedance are studied and discussed.

An important aspect of the arterial blood flow is that it remains mostly an entry flow. With this in view the blood flow in the entry region in the presence of a stenosis is studied in Chapter III. The flow field is divided into two regions - a core and a boundary layer. As in the classical boundary layer problem, the equations in the two regions are solved separately using a matching condition at the edge of the boundary layer. For different stenotic geometries, the velocity, shearing stress and vorticity are computed and the phenomenon of boundary layer separation is discussed.

In Chapter IV, the swirling entry flow in a rigid tube with a mild stenosis at the entrance is studied using the method of matched asymptotic expansions. As in

Chapter III, the flow is considered to be divided into two regions, (1) an inviscid core and (2) a thin boundary layer. The swirl velocity is prescribed at the entry alongwith the longitudinal flow. The reduction in blood supply due to the presence of stenosis and shear-stress on the walls are presented for different values of the swirl parameter.

In Chapter V, the abnormality in the pulsatile flow conditions due to the presence of stenosis is studied by modelling the artery as a rigid circular tube. Special attention is given to the phenomenon of the formation of a steady-streaming flow, induced by the Reynolds stresses in the oscillatory boundary layer. Analytical solutions are obtained for the velocity and shear-stress. The combined effects of stenosis and pulsatility of the flow are studied.

Curvature forms an important feature of the thoracic aorta and Chapter VI deals with the fully developed flow of fluid in a curved pipe of non-uniform curvature under a pressure gradient. The cross-section of the pipe is an ellipse, the axes of which are in an arbitrary position with respect to the radius of curvature of the pipe. The flow is assumed to be steady and at low Reynolds number. The method of solution involves successive approximation with respect to the Dean number. Effects of asymmetry and non-uniformity of the curvature on the wall shear stress are discussed.

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