

**ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF  
INTERACTION PROBLEMS WITH DENSE SAND  
CONSIDERING THE EFFECT OF STRESS-PATH**

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CERTIFICATE

This is to certify that the thesis entitled 'ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF INTERACTION PROBLEMS WITH DENSE SAND CONSIDERING THE EFFECT OF STRESS-PATH' submitted by Mr R.Kuberan to the Indian Institute of Technology, Delhi for the award of degree of DOCTOR OF PHILOSOPHY is a record of the bonafide research work carried out by him. Mr Kuberan has worked under our supervision for the submission of this thesis, which, to our knowledge, has reached the requisite standard.

The thesis, or any part thereof has not been submitted to any other University or Institution for the award of any Degree or Diploma.



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TO MY MOTHER

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ABSTRACT

Finite element method is being increasingly used in the analysis of many geotechnical engineering problems. The importance of modelling the behaviour of the soil is well recognised and a number of constitutive models are being developed taking into account various factors which influence the behaviour of the soil. The non-linear elastic analysis does not account for the dilation of soil during shearing. The constitutive models using elasto-plastic or elasto-viscoplastic approaches do not consider the effect of stress path and initial stress condition correctly. It has been well established that these factors very much influence the constitutive behaviour of soil, especially in the case of dense sand. Therefore, a constitutive model for dense sand considering the effect of stress-path and insitu stress condition has been developed following initial strain approach for use in elasto-plastic as well as elasto-viscoplastic methods.

In developing the constitutive model, the effect of any stress change is considered to consist of the effects due to consolidation and shear. The change in stress level is reckoned with respect to the initial stress condition for calculating the plastic strains. Thus the stress-path and initial stress condition are properly accounted for. The soil parameters are evaluated from a consolidation rebound test and two pure shear tests directed above and below the consolidation line. These tests are also conducted following the consolidation line corresponding to the initial stress condition.

The constitutive model developed has been incorporated in a computer program considering a single stress point and triaxial tests along a variety of stress-paths have been analysed numerically. The predictions have been compared with available experimental data. It is seen that the model is capable of predicting the constitutive behaviour along all the stress paths satisfactorily.

The constitutive model has been incorporated in Finite Element program and typical boundary value problems have been analysed using this model. First, the active earth pressure problem involving uniform stress-paths has been taken. The results have been compared with the

classical theory and non-linear elastic analysis. It is observed that the constitutive relationship predicts volume change in accordance with the stress-paths followed.

The footing problem which involves varied stress-paths has been analysed. It is shown that the analysis using the model is capable of predicting the behaviour of dense sand in this case also. Further, the effect of stiffness and embedment in the interaction of the footing soil system have been studied.

The nature of load-settlement relationship is found to be linear. The nature of contact pressure distribution is influenced by the stiffness of the system for a depth of embedment. The stiffness of the footing-soil system changes with embedment and accordingly the contact pressure also changes. The resistance of the soil also influences the contact pressure distribution. Varied nature of stress-paths is observed in various zones of the foundation medium. The effect of embedment on the stress-paths is considerable for the zones away from the footing. The magnitudes of displacements, volumetric and shear strains are also affected by the embedment.

The predictions from the analysis have also been compared with available experimental results. It is observed that the prediction is comparable with experimental results reasonably well.

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