FINITE-AMPLITUDE STABILITY OF PIPE-POISEUILLE FLOW TO THREE-DIMENSIONAL NON-AXISYMMETRIC DISTURBANCES

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THESIS
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"DEDICATED TO MY PARENTS"

CERTIFICATE

This is to certify that the thesis entitled "FINITE-AMPLITUDE STABILITY OF PIPE-POISEUILLE FLOW TO THREE-DIMENSIONAL NON-AXISYMMETRIC DISTURBANCES" by Mr. Subhasis Maji, submitted to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him under my supervision and guidance. The work in this thesis, in my opinion, has reached the standard fulfilling the requirements for the award of the degree of Doctor of Philosophy. The research reports and results presented in this thesis have not been submitted in part or in full to any other University or Institute, for the award of any degree or diploma.

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ABSTRACT

The object of the present work is to study the stability of pipe-Poiseuille flow to finite-amplitude non-axisymmetric disturbances by calculating higher order Landau-coefficients of the Stuart-Watson (1960) formalism. The specific theory used was the equilibrium-amplitude formulation of Reynolds and Potter (1967).

It is known from the earlier analytical works that according to linear theory, pipe-Poiseuille flow is least stable to a non-axisymmetric mode of disturbance which has an azimuthal wavenumber of $n = 1$. In the present work, we studied the non-linear stability of pipe-Poiseuille flow, for this particular least-stable mode of disturbance.

The Stuart-Landau series was analysed by means of the Shanks (1955) method, to investigate the possible existence of equilibrium states. Also, the radius of convergence of the Stuart-Landau series was calculated by using Domb-Sykes plots. Within the radius of convergence, the value of the amplitude $|A|$ which made the correct sum of the Stuart-Landau series as zero, was determined, and this value of $|A|$ is actually the equilibrium amplitude.

Results show that there is a preferred band of the spatial wavenumber α (i.e. $1 < \alpha \leq 4$), where

threshold-amplitudes or equilibrium-amplitudes exist. It is also found that the value of the minimum equilibrium-amplitude corresponding to a given Reynolds number R, i.e. $(A_e)_{min}$, increases with decreasing values of the Reynolds number R. Particularly, there is a sharp increase in $(A_e)_{min}$ for $R < 2000$. This is in qualitative agreement with the value of the critical Reynolds number known in engineering practice, where pipe flow is known to become turbulent for $R > 2000$, but remains laminar for $R < 2000$.

The present results also establish the usefulness of the Stuart-Watson formalism, in the study of the stability of pipe-Poiseuille flow.

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