

A THESIS ON
STRESS DISTRIBUTION
IN
CAMs AND OTHER PROFILED PLATES

By
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C E R T I F I C A T E

This is to certify that the thesis entitled, 'Stress Distribution in Cams and Other Profiled Plates' which is being submitted by Mr. A. M. Hardas to the Indian Institute of Technology, Delhi for the award of the Degree of Doctor of Philosophy in Applied Mechanics, is a bonafide research work carried out by him during the last two years and ten months, under my guidance and supervision. His thesis has reached the standard of fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full, to any other university or institution for the award of any degree or diploma.



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A C K N O W L E D G E M E N T S

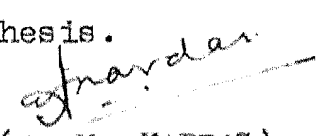
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S U M M A R Y

The problem of the distribution of stresses in cams and other profiled plates, when subjected to static load or to high speed rotation about an axis perpendicular to the plane of the plate, has been considered in this investigation. The solution of the problem for a rotating circular elastic disc with or without concentric circular hole is well known. In the last two decades, solutions have been obtained for simply connected plates with simple edge profiles, e.g., triangle, square, ellipse etc. In these problems the boundary conditions have been simplified to facilitate analytical solutions. These solutions are still too complex to be directly useful. The cams and other machine components, in general, have intricate shapes because they are designed on the basis of kinematic requirements. In practice, since they are mounted on a shaft, they cannot be treated as simply connected. Moreover boundary conditions are more complex than assumed in the reported analytical solutions.

Therefore numerical and experimental techniques have been used in this investigation on plates of arbitrary shape subjected to various boundary conditions. As in other cases, we consider the problem in the domain of linear elasticity. The numerical technique used is

the finite element method.. We have used two dimensional square and triangular elements and followed Pian's hybrid element approach. A computer program has been developed and tested on profiles for which analytical solutions are available, e.g., a rotating circular disc. The mesh has been refined near the centre in order to obtain more detailed distribution of the stresses in that region. Next we have considered the case of an elliptical plate for which the analytical results were given by Mitchell and Warren (44), using Muskhelishvili's complex variables technique. Again a computer program was developed to solve some cases using the equations given by Mitchell and Warren. In all cases the finite element solutions agree well with the analytical solutions. Since the finite element computer program is for general profile and boundary conditions, the numerical solutions for the following cases have been obtained and plotted in convenient forms:

1. eccentric solid disc,
2. elliptical plate having a circular hole in the centre,
3. eccentric discs mounted on a rigid and loose shafts,
4. tangent cam with a hole and a key-slot.

Similarly we have considered the stresses in the cam due to rotational acceleration which is of great practical importance. This aspect of the problem has completely escaped the notice of the previous investigators in this

field. Finally we have considered the problem of cam of a common shape, e.g., a tangent cam with a key-slot for which the combined stress distribution due to oblique static load at its tip, uniform rotation and acceleration has been obtained.

The numerical solutions for stresses due to uniform rotation have been supplemented by experimental investigations. The 'Frozen Stress' oven and a rotation arrangement were specially designed and fabricated for this purpose. Since the type of constraints at the inner boundary influences greatly the distribution of stresses around the hole, we have examined different constraints at the inner boundary experimentally.

This analysis is intended to be useful in the design of cams and other rotating components since it gives information about the magnitude and location of maximum shear or principal stresses. One can also determine the maximum speed beyond which, when rotated, the components are deformed plastically. The information about the direction of principal stresses is needed for providing proper stiffeners or reinforcements whenever required.

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