

COMPREHENSIVE ACCESSIBILITY OF EQUATIONS BY VISUALLY IMPAIRED

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**AMAR NATH AND SHASHI KHOSLA SCHOOL OF INFORMATION
TECHNOLOGY
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by

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AMAR NATH AND SHASHI KHOSLA SCHOOL OF INFORMATION
TECHNOLOGY

Submitted

In Fulfilment of the Requirements of the Degree of

DOCTOR OF PHILOSOPHY

to the



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to
my family

CERTIFICATE

This is to certify that the thesis entitled “**COMPREHENSIVE ACCESSIBILITY OF EQUATIONS BY VISUALLY IMPAIRED**” being submitted by **Mr. Akashdeep Bansal** to the Indian Institute of Technology Delhi in fulfilment of the requirements for the award of the degree of **Doctor of Philosophy** is a record of Bonafide research work performed by him under my guidance and supervision at Amar Nath and Shashi Khosla School of Information Technology, Indian Institute of Technology Delhi, India. The results obtained herein have not been submitted in part or in full to any other university or institute for the award of any degree to the best of my knowledge.



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Abstract

While blind and visually impaired (BVI) readers can access digital text information using auditory interface through screen readers, accessing mathematical equations is much more challenging task, which is known to limit access to STEM subjects for BVI students. This is primarily due to the complexities of equation layout as well as the linear syntactic rendering. Consequently, even the relatively simpler equations (containing small number of symbols) can lead to a very lengthy and unnatural speech string resulting in a high cognitive load for the reader. This thesis aims to address this issue by proposing innovative solutions to improve equation comprehension and accessibility for BVI individuals.

One approach to reduce this load can be to abstract parts of an equation/expression via some abbreviating symbol which is then read out separately. This division/partitioning into expressions is based on computing the cognitive complexity as "felt" by the reader. The unnaturalness in the speech string is due to syntactic rendering. Which can be improved by contextually aware audio rendering of equations- Adapting the speech output based on the context available in the surrounding text.

The first part of the research focuses on identifying a suitable metric for measuring the cognitive complexity of equations. A comprehensive user study involving eighteen BVI participants was conducted to capture their responses, including equation reproduction errors and time taken for comprehension. This data is used to propose candidate metrics and establish correlations with existing complexity measures in open-source math accessibility systems. Ultimately, a reliable complexity metric was identified that aligned well with the user study results, offering potential integration into screen readers for equation accessibility.

The second part of the thesis addresses the contextually aware audio rendering. We worked on an algorithm to extract contextual semantics of equations based on local definitions. The algorithm comprises of multiple modules, and utilizes various machine learning and pattern matching techniques. This contextual understanding enables adaptive audio rendering that takes into account the surrounding text, improving the comprehension and usability of equations.

This research contributes to the advancement of accessibility in STEM education for visually impaired individuals, particularly in the realm of equation comprehension and audio rendering. By introducing a cognitive complexity metric and algorithm for contextual semantic analysis, this thesis offers practical solutions to make mathematical equations more accessible to BVI readers. These findings have the potential to significantly impact the inclusion and educational opportunities for visually impaired students, empowering them to pursue STEM subjects with greater confidence and success.

सारांश

जबकि नेत्रहीन और दृष्टिबाधित (बी. वी. आई.) पाठक स्क्रीन रीडर के माध्यम से श्रवण इंटरफेस का उपयोग करके डिजिटल सामग्री को पढ़ सकते हैं, गणितीय समीकरणों को समझना बहुत अधिक चुनौतीपूर्ण कार्य है, जो बी. वी. आई. छात्रों के लिए विज्ञान, प्रौद्योगिकी, अभियांत्रिकी और गणित (एस. टी. ई. एम.) विषयों तक की पहुँच को सीमित करता है। यह मुख्य रूप से समीकरण के विन्यास की जटिलताओं के साथ-साथ रैखिक वाक्य रचना प्रतिपादन के कारण है। नतीजतन, अपेक्षाकृत सरल समीकरण (जिनमें कम संख्या में चिन्ह होते हैं) भी एक बहुत लंबी और अप्राकृतिक भाषण श्रृंखला का कारण बन सकते हैं जिसके परिणामस्वरूप पाठक के लिए एक उच्च संज्ञानात्मक भार हो सकता है। इस शोध प्रबंध का उद्देश्य बी. वी. आई. व्यक्तियों के लिए समीकरण की समझ और पहुँच में सुधार के लिए नवीन समाधानों का प्रस्ताव करके इस मुद्दे को संबोधित करना है।

इस बोझ को कम करने का एक तरीका यह हो सकता है कि समीकरण/अभिव्यक्ति के कुछ भागों को किसी संक्षिप्त चिन्ह के माध्यम से सारांशित किया जाए, जिसे बाद में अलग से पढ़ा जाए। अभिव्यक्तियों में यह विभाजन पाठक द्वारा महसूस की गई संज्ञानात्मक जटिलता की गणना पर आधारित होना चाहिए। भाषण श्रृंखला में अप्राकृतिकता वाक्य रचना प्रतिपादन के कारण होती है। जिसे समीकरणों के संदर्भ-संवेदनशील ध्वनि प्रस्तुति (आसपास के पाठ में उपलब्ध संदर्भ के आधार पर भाषण उत्पादन को अनुकूलित करना) द्वारा सुधारा जा सकता है।

शोध का पहला भाग समीकरणों की संज्ञानात्मक जटिलता को मापने के लिए एक उपयुक्त मापक की पहचान करने पर केंद्रित है। अठारह बी. वी. आई. प्रतिभागियों को शामिल करते हुए एक व्यापक उपयोगकर्ता अध्ययन उनकी प्रतिक्रियाओं को पकड़ने के लिए आयोजित किया गया था, जिसमें समीकरण प्रजनन त्रुटियाँ और समझ के लिए लिया गया समय शामिल था। इस डेटा का उपयोग उम्मीदवार मापक का प्रस्ताव करने और ओपन-सोर्स गणित अभिगम्यता प्रणालियों में मौजूदा जटिलता उपायों के साथ सहसंबंध स्थापित करने के लिए किया गया है। अंततः, एक विश्वसनीय जटिलता मापक की पहचान की गई जो उपयोगकर्ता अध्ययन परिणामों के साथ अच्छी तरह से संरेखित है, जो समीकरण की पहुँच के लिए स्क्रीन रीडर में संभावित एकीकरण की पेशकश करता है।

शोध प्रबंध का दूसरा भाग संदर्भ-संवेदनशील ध्वनि प्रस्तुति को संबोधित करता है। हमने स्थानीय परिभाषाओं के आधार पर समीकरणों के प्रासंगिक शब्दार्थ को निकालने के लिए एक एल्गोरिथ्म पर काम किया। इस एल्गोरिथ्म में कई मॉड्यूल शामिल हैं, जो विभिन्न मशीन लर्निंग और पैटर्न मिलान तकनीकों का उपयोग करते हैं। यह संदर्भगत समझ ऐसी अनुकूली ध्वनि प्रस्तुति को सक्षम बनाती है, जो आसपास की सामग्री को ध्यान में रखते हुए प्रस्तुत की जाती है, जिससे समीकरणों की बोधगम्यता और उपयोगिता में सुधार होता है।

यह शोध दृष्टिबाधित व्यक्तियों के लिए, विशेष रूप से समीकरण बोध और ध्वनि प्रस्तुति के क्षेत्र में, एस. टी. ई. एम. शिक्षा में पहुँच की उन्नति में योगदान देता है। प्रासंगिक शब्दार्थ विश्लेषण के लिए एक संज्ञानात्मक जटिलता मापक और एल्गोरिथ्म पेश करके, यह शोध प्रबंध गणितीय समीकरणों को बी. वी. आई. पाठकों के लिए अधिक सुलभ बनाने के लिए व्यावहारिक समाधान प्रदान करता है। इन निष्कर्षों में दृष्टिबाधित छात्रों के लिए समावेश और शैक्षिक अवसरों को महत्वपूर्ण रूप से प्रभावित करने की क्षमता है, जिससे वे अधिक आत्मविश्वास और सफलता के साथ एस. टी. ई. एम. विषयों में आगे बढ़ने के लिए सशक्त हो सकते हैं।

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