

**ANALYTICAL AND NUMERICAL STUDIES OF LASER
WAKEFIELD ACCELERATION IN BUBBLE REGIME**

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**ANALYTICAL AND NUMERICAL STUDIES OF LASER
WAKEFIELD ACCELERATION IN BUBBLE REGIME**

by

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Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



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Dedicated to

my family and loved ones

&

those

whoever inspired and encouraged me

Certificate

This is to certify that the thesis entitled “**Analytical and Numerical Studies of Laser Wakefield Acceleration in Bubble Regime**” being submitted by **Mr. Sonu Kumar** is worthy of consideration for the award of the degree of Doctor of Philosophy and is a record of the original bonafide research work carried out by him under our guidance and supervision, and that the results contained in it have not been submitted in part or full to any other university or institute for award of any degree / diploma.

We certify that he has pursued the prescribed course of research. We approve the thesis for the award of the degree of Doctor of Philosophy.



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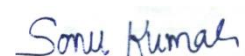
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(Sonu Kumar)

Abstract

Plasma wakefield acceleration is a promising technique to build compact and powerful particle accelerators. In such accelerators, the electric fields required to accelerate charged particles are sustained by electron density modulations in the plasma which may be excited by a short intense laser pulse propagating through the plasma. The radiation pressure of the laser pushes away the plasma electrons which are pulled back by ions which are considered to be stationary due to their heavier mass. This initiates a charge density oscillation behind the laser pulse referred to as plasma wakefield, similar to the wake of a boat in a river. A bunch of electrons injected into this wakefield are constantly pulled towards the pulse and get accelerated to a good fraction of the speed of light very quickly. This scheme of particle acceleration is known as Laser-Plasma Wakefield Acceleration (LWFA). An ultra-intense ($> 10^{18} \text{ W/cm}^2$) laser pulse having a pulse length shorter than the plasma wavelength expels the electrons from a region due to the trapping of electromagnetic energy and creates ion cavities or bubbles. The expelled plasma electrons are accumulated around the bubble surface and some of them are self-injected into the wakefield and get accelerated. The bubble regime of electron acceleration has the advantage that no external electron injection mechanism is needed as the electrons are self-injected and also no preformed plasma channel is required for the electron acceleration since the bubbles can guide the laser pulse up to many Rayleigh lengths.

This thesis describes both analytical and numerical studies for LWFA in the bubble regime. Various shapes of bubbles such as spherical, longitudinal-ellipsoid or transverse-ellipsoid are found to be controlled by d'Alembert equations under three different Gauge conditions. For each of the Gauge conditions, wakefield potential inside various shapes of the bubbles has been calculated analytically and is applied to numerically investigate the self-injection of plasma electrons in the bubble. Several parameters such as bubble velocity, bubble radius and impact parameters are described and their effect on bubble formation and self-injection of electrons have been studied numerically solving the coupled differential equations. Particle-In-Cell (PIC) simulations have been performed to numerical model the experimental conditions for LWFA to understand various complex kinetic and relativistic phenomena of laser-plasma interactions which have not been addressed experimentally. The simulations have been performed using the PIC code *Smilei* which is a high-performance open-source code for the simulation of relativistic laser-plasma interaction. The focus of these simulations lies in the optimization of laser pulse specifications and plasma density profiles for efficient particle acceleration in the bubble regime of LWFA. In this regard, the study of wakefield generation by the modulation of various plasma density profiles has been presented. To benchmark the

experimental conditions, a comparison has been presented for various temporal and spatial profiles of laser vis-à-vis the plasma density modulation, wakefield strength, electrons self-injection, energy spectrum and kinetic energy gradient of accelerated electrons. The scheme of multi-pulse laser wakefield acceleration (MP-LWFA) has been investigated and the merits of the multi-pulse LWFA over single-pulse LWFA has been demonstrated. The work described in this thesis attempts to benchmark the experimental conditions required to explore various aspects of laser plasma wakefield acceleration.

प्लाज्मा वेकफील्ड त्वरण कॉम्पैक्ट और शक्तिशाली कण त्वरक बनाने के लिए एक आशाजनक तकनीक है। ऐसे त्वरक में, आवेशित कणों को गति देने के लिए आवश्यक विद्युत क्षेत्र प्लाज्मा में इलेक्ट्रॉन संख्या घनत्व मॉड्युलेशन द्वारा बनाए रखा जाता है जो प्लाज्मा के माध्यम से फैलने वाली एक छोटी तीव्र लेजर पल्स द्वारा उत्तेजित हो सकता है। लेजर का विकिरण दबाव प्लाज्मा इलेक्ट्रॉनों को दूर धकेलता है जो आयनों द्वारा वापस खींच लिए जाते हैं जिन्हें उनके भारी द्रव्यमान के कारण स्थिर माना जाता है। यह प्लाज्मा वेकफील्ड के रूप में संदर्भित लेजर पल्स के पीछे एक चार्ज घनत्व दोलन शुरू करता है, जो नदी में नाव के चलने के समान होता है। इस वेकफील्ड में प्रेषित किए गए इलेक्ट्रॉनों का एक गुच्छा लगातार पल्स की ओर खींचा जाता है और बहुत तेज़ी से प्रकाश की गति के एक अच्छे अंश तक त्वरित हो जाता है। कण त्वरण की इस योजना को लेजर-प्लाज्मा वेकफील्ड त्वरण के रूप में जाना जाता है। एक अति तीव्र ($> 10^{18}$ वॉट/सेमी²) लेजर पल्स, जिसकी स्पंद लंबाई प्लाज्मा तरंग दैर्ध्य से कम होती है, विद्युत चुम्बकीय ऊर्जा के फंसने के कारण एक क्षेत्र से इलेक्ट्रॉनों को बाहर निकाल देता है और आयन गुहा या बबल बनाता है। निष्कासित प्लाज्मा इलेक्ट्रॉन बबल की सतह के चारों ओर जमा होते हैं और उनमें से कुछ को वेकफील्ड में स्व-प्रेषित किया जाता है और त्वरित हो जाता है। इलेक्ट्रॉन त्वरण के बबल व्यवस्था का लाभ यह है कि किसी बाहरी इलेक्ट्रॉन प्रेषण तंत्र की आवश्यकता नहीं होती है क्योंकि इलेक्ट्रॉन स्वयं प्रेषित होते हैं और इलेक्ट्रॉन त्वरण के लिए किसी पूर्वनिर्मित प्लाज्मा चैनल की भी आवश्यकता नहीं होती है क्योंकि बबल लेजर पल्स को कई रैले लंबाई तक निर्देशित कर सकते हैं।

यह शोध-ग्रंथ बबल क्षेत्र में लेजर वेकफील्ड त्वरण के लिए विश्लेषणात्मक और संख्यात्मक दोनों अध्ययनों का वर्णन करती है। बबल के विभिन्न आकार जैसे गोलाकार, अनुदैर्ध्य-दीर्घवृत्ताभ या अनुप्रस्थ-दीर्घवृत्ताभ तीन अलग-अलग गेज स्थितियों के तहत डी'अलेम्बर्ट समीकरणों द्वारा नियंत्रित पाए जाते हैं। गेज स्थितियों में से प्रत्येक के लिए, बबल के विभिन्न आकारों के अंदर वेकफील्ड क्षमता की गणना विश्लेषणात्मक रूप से की गई है और इसे बबल में प्लाज्मा इलेक्ट्रॉनों के स्व-प्रेषण की संख्यात्मक रूप से जांच करने के लिए लागू किया गया है। बबल वेग, बबल त्रिज्या और प्रभाव मापदंडों जैसे कई मापदंडों का वर्णन किया गया है और बबल गठन और इलेक्ट्रॉनों के स्व-प्रेषण पर उनके प्रभाव का संख्यात्मक रूप से युग्मित अंतर समीकरणों को हल करने का अध्ययन किया गया है। पार्टिकल-इन-सेल (पीआईसी) सिमुलेशन लेजर-प्लाज्मा इंटरैक्शन के विभिन्न जटिल गतिज और सापेक्ष घटनाओं को समझने के लिए लेजर वेकफील्ड त्वरण के लिए प्रायोगिक स्थितियों के संख्यात्मक मॉडल के लिए किए गए हैं जिन्हें प्रयोगात्मक रूप से संबोधित नहीं किया गया है। पीआईसी कोड स्माइली का उपयोग करके

सिमुलेशन का प्रदर्शन किया गया है जो सापेक्षवादी लेजर-प्लाज्मा मिलाप के सिमुलेशन के लिए एक उच्च-प्रदर्शन खुला-स्रोत कोड है। इन सिमुलेशन का ध्यान लेजर वेकफील्ड त्वरण के बबल क्षेत्र में कुशल कण त्वरण के लिए लेजर पल्स विनिर्देशों और प्लाज्मा घनत्व प्रोफाइल के अनुकूलन में निहित है। इस संबंध में, विभिन्न प्लाज्मा घनत्व प्रोफाइल के मॉड्युलेशन द्वारा वेकफील्ड उन्मूलन का अध्ययन प्रस्तुत किया गया है। प्रयोगात्मक स्थितियों को बेंचमार्क करने के लिए, प्लाज्मा घनत्व मॉड्युलेशन, वेकफील्ड परिमाण, इलेक्ट्रॉन स्व-प्रेषण, ऊर्जा वर्णक्रम और त्वरित इलेक्ट्रॉनों की गतिज ऊर्जा प्रवणता की तुलना में लेजर के विभिन्न लौकिक और स्थानिक प्रोफाइल के लिए एक तुलना प्रस्तुत की गई है। बहू-पल्स लेजर वेकफील्ड त्वरण की योजना की जांच की गई है और एक-पल्स लेजर वेकफील्ड त्वरण पर बहू-पल्स लेजर वेकफील्ड त्वरण की खूबियों का प्रदर्शन किया गया है। इस शोध-ग्रंथ में वर्णित कार्य लेजर प्लाज्मा वेकफील्ड त्वरण के विभिन्न पहलुओं का पता लगाने के लिए आवश्यक प्रायोगिक स्थितियों को बेंचमार्क करने का प्रयास किया गया है।

Table of Contents

Certificate.....	i
Acknowledgements.....	ii
Abstract.....	iii
Table of Contents.....	vii
List of Figures.....	xi
List of Symbols.....	xvi
List of Tables.....	xx
Chapter 1: Introduction and Literature Review.....	1
1.1. Particle Accelerators.....	1
1.2. Plasma Accelerators.....	1
1.2.1. Different types of plasma accelerators.....	2
1.3. Laser Wakefield Acceleration.....	2
1.3.1. Plasma Oscillations.....	3
1.3.2. Laser.....	4
1.3.3. Linear wakefield acceleration.....	5
1.3.4. Nonlinear wakefield acceleration.....	5
1.3.5. Bubble wakefield acceleration.....	7
1.4. Numerical Modelling.....	7
1.5. PIC Algorithms.....	7
1.6. Literature Review.....	8
1.7. Motivation.....	10
1.8. Thesis Organisation.....	10
Chapter 2: Different Gauges in Bubble Wakefield Acceleration.....	15
2.1. Dephasing length in bubble regime	15
2.2. Analytical Investigation.....	16
2.2.1. $A_x = \phi$ and generated wakefield potential $\Phi = A_x + \phi$	17

2.2.1.1. Bubble geometry.....	18
2.2.1.2. Wakefield potential in different shaped bubble regime.....	19
2.2.1.3. Electromagnetic field in the bubble regime.....	20
2.2.1.4. Accelerator gradient.....	21
2.2.1.5. Maximum energy gain.....	21
2.2.2. $\vec{\nabla}_{\perp} \cdot \vec{A}_{\perp} = -\frac{\partial \phi}{\partial \zeta}$, $\Phi = A_x - \phi$	22
2.2.2.1. Bubble geometry.....	23
2.2.2.2. Wakefield potential	24
2.2.2.3. Accelerating gradient.....	24
2.2.2.4. Maximum energy gain.....	25
2.3. Special case.....	26
2.3.1. Electromagnetic field in bubble regime.....	26
2.4. Conclusions.....	28
Chapter 3: Electrons Self-injection in Different Shaped Bubbles	29
3.1. Numerical Investigations.....	29
3.2. Results and Discussion.....	30
3.2.1. Shape of the bubble.....	31
3.2.2. Trajectory of self-injected plasma electrons.....	32
3.2.3. Longitudinal phase-space of self-injected plasma electrons	32
3.2.4. Transverse phase-space of self-injected plasma electrons	32
3.2.5. Effect of impact parameter on electron trajectory.....	33
3.2.6. Effect of bubble radius on electron trajectory.....	35
3.2.7. Effect of bubble velocity.....	36
3.3. Concluding Remarks.....	38
Chapter 4: Role of Ultrashort Trapezoidal Temporal Pulse Profile in Bubble Wakefield Acceleration	39
4.1. Simulation Method and Parameters.....	39
4.2. Laser Pulse Profile.....	40
4.3. Effect of Laser Pulse Profile on Wakefield	42
4.4. Effect of Laser Pulse profile on Electron Density Variation	42
4.5. Effect of Laser Profile on Electron Energy Spectra	43
4.6. Kinetic Energy Gradient of Accelerated Electrons.....	45

4.7. Summary and Conclusions.....	46
Chapter 5: Ultrashort Single-pulse and Multi-pulse Driven Laser Wakefield Acceleration.....	47
5.1. Numerical Modelling.....	47
5.2. Simulation Parameters.....	48
5.3. Results and Discussion.....	50
5.4. Summary and Conclusions.....	55
Chapter 6: Trapezoidal and Polygon Density Regions in Bubble Wakefield Acceleration.....	57
6.1. Details of Simulation Method and Parameters.....	57
6.2. Trapezoidal Density Profile.....	58
6.2.1. Variation of longitudinal electric field E_x	59
6.2.2. Electron density modulation.....	59
6.2.3. Energy spectrum.....	60
6.2.4. Kinetic energy.....	61
6.3. Polygonal Density Profile.....	63
6.3.1. Electron density modulation and longitudinal electric field.....	63
6.3.2. Probe diagnostic of electron density and longitudinal electric field.....	64
6.3.3. Energy spectrum and kinetic energy gradient.....	64
6.4. Conclusions.....	65
Chapter 7: Bubble Formation and Electrons Self-injection in Colliding Laser Wakefield Acceleration.....	67
7.1. Simulation Details.....	67
7.2. Mechanism of Colliding Pulse Scheme.....	68
7.3. Pump Pulse and Injection Pulse System.....	69
7.4. Wakefield Generation.....	69
7.5. Electron Density Modulation.....	72
7.6. Energy Spectrum of Accelerated Electrons.....	74
7.7. Conclusions.....	75
Chapter 8: Conclusions, Application and Future Scope of Thesis Work.....	77
8.1. Conclusions of Thesis Work.....	77

8.1.1. Bubble geometry.....	77
8.1.2. Self-injection of plasma electrons.....	77
8.1.3. Role of temporal trapezoidal laser pulse profile.....	78
8.1.4. Bubble wakefield by train of laser pulses.....	78
8.1.5. Role of trapezoidal and polygon plasma density profile.....	78
8.1.6. Colliding laser pulse scheme.....	79
8.2. Application of Thesis Work.....	79
8.3. Future Scope of Thesis Work.....	80
References.....	83
Brief Bio-Data of Author.....	89

List of Figures

Figure 1.1: Schematic diagram of laser wakefield acceleration.

Figure 1.2: Intensity vs propagation direction along x-direction and polarized along y-direction.

Figure 1.3: Intensity vs time.

Figure 1.4: Laser electric field E_y polarized in y -direction and propagating from left to right along x-direction and wakefield E_x propagating in x-direction at time $t = 0.4$ ps.

Figure 1.5: Longitudinal electric field E_x and transverse electric field E_y at time $t = 0.5$ ps for laser pulse strength $a_0 = 5.2$.

Figure 1.6: Electrons motion inside formed bubble regime.

Figure 1.7: PIC loop.

Figure 2.1 Variation of geometrical parameter η^2 along y-direction related to shape of bubble having residual electrons density n_a along x-direction with a change of transverse geometrical coefficient C.

Figure 2.2: Variation of wakefield potential $\Phi = \left(-\frac{1}{4} - C\right)\zeta^2 - \frac{y^2}{4}$ along (ζ, y) direction for different values of C.

Figure 2.3: Variation of accelerating field or wakefield E_x with bubble radius r_b along the propagation direction with different values of transverse geometrical coefficient C. In unnormalized form, the value of $E_0 = 96$ GV/m for $\omega_p = \sqrt{4\pi n_0 e^2 / m_e} = 5.656 \times 10^{13}$ rad/s and $n_0 = 10^{18}$ cm⁻³.

Figure 2.4: Variation of maximum energy gain W_{max} with relativistic Lorentz factor γ_p along the x-direction for different values of geometrical coefficient C.

Figure 2.5: Variation of geometrical parameter η^2 as a function of residual electrons density n_a for different transverse coefficient C.

Figure 2.6: Variation of wakefield potential $\Phi = \left(\frac{1}{2} - C\right)\zeta^2 + \frac{y^2}{4}$ as a function of (ζ, y) direction with changing of transverse coefficient C.

Figure 2.7: Variation of accelerating field or wakefield E_x as a function of bubble radius r_b along propagation direction x, for different values of transverse geometrical coefficient C.

Figure 2.8: Variation of maximum energy gain W_{max} along y-direction with relativistic Lorentz factor γ_p along x-direction with the change of transverse geometrical coefficient C.

Figure 2.9: Variation of wakefield potential $\Phi = \left(\frac{1}{4} - C\right) \zeta^2 + \frac{y^2}{4}$ along (ζ, y) direction with changing of transverse coefficient C.

Figure 2.10: Variation of accelerating field or wakefield E_x along y – direction with bubble radius r_b along propagation direction x- axis with the change of transverse geometrical coefficient C.

Figure 2.11: Variation of maximum energy gain W_{max} along y-direction with relativistic Lorentz factor γ_p along x-direction with the change of transverse geometrical coefficient C.

Figure 3.1: This figure shows the parameters related with bubble and $R = \sqrt{\zeta_0^2 + \rho^2}$ is bubble radius, where ζ_0 and $y_0 = \rho$ are longitudinal and transverse coordinates.

Figure 3.2: Trajectory of electrons with variation of shapes of the bubble with $C = 0$ (spherical bubble), $C = -0.2$ (transverse ellipsoid bubble) and $C = 0.2$ (longitudinal ellipsoid bubble) when $p_{x0} = 0, p_{y0} = 0, \zeta_0 = \sqrt{R^2 - y_0^2}, y_0 = \rho = 10$ and bubble velocity $V = 0.969$ for Lorentz factor $\gamma_0 = 4$. Here radius of the bubble $R = 10$.

Figure 3.3: Longitudinal phase-space diagram of trapped electrons in different shaped bubble with $C = 0$ for spherical bubble (a), with $C = 0.2$ for longitudinal ellipsoid bubble (b) and with $C = -0.2$ for transverse ellipsoid bubble (c), when $V = 0.969$ for $\gamma_0 = 4$ with impact parameter of the trapped electron as $\rho = 10$ and radius of the bubble as $R = 10$.

Figure 3.4: Transverse phase-space diagram of the trapped electrons in different shaped bubble when $C = 0$ (spherical bubble) in (a), $C = 0.2$ (longitudinal ellipsoid bubble) in (b) and $C = -0.2$ (transverse ellipsoid bubble) in (c) $V = 0.969$ for $\gamma_0 = 4$ with impact parameter of the trapped electron as $\rho = 10$ and radius of the bubble as $R = 10$.

Figure 3.5: Trajectory of the electrons in spherical bubble at $C = 0$ and with initial electrons momentum value $p_{x0} = 0, p_{y0} = 0, \zeta_0 = \sqrt{R^2 - y_0^2}$, and $V = 0.969$ for Lorentz factor $\gamma_0 = 4$. Here radius of the bubble is $R = 10$ with different values of impact parameter $y_0 = \rho$.

Figure 3.6: Trajectory of the electrons in longitudinal ellipsoid bubble at $C = 0.2$ and with initial electrons momentum value $p_{x0} = 0, p_{y0} = 0, \zeta_0 = \sqrt{R^2 - y_0^2}$, and $V = 0.969$ for Lorentz factor $\gamma_0 = 4$. Here radius of the bubble is $R = 10$ with different values of impact parameter $y_0 = \rho$.

Figure 3.7: Trajectory of the electrons in transverse ellipsoid bubble at $C = 0.2$ and with initial electrons momentum value $p_{x0} = 0, p_{y0} = 0, \zeta_0 = \sqrt{R^2 - y_0^2}$, and $V = 0.969$ for Lorentz factor $\gamma_0 = 4$. Here radius of the bubble is $R = 10$ with different values of impact parameter $y_0 = \rho$.

Figure 3.8: Trajectory of the electrons in spherical bubble at $C = 0.0$ and with bubble velocity $V = 0.969$ for Lorentz factor $\gamma_0 = 4$. Here the impact parameter is $\rho = 6$ with different values of bubble radius $R_1 = 10, R_2 = 8, R_3 = 6$.

Figure 3.9: Trajectory of the electrons in longitudinal ellipsoid bubble at $C = 0.2$ and with bubble velocity $V = 0.969$ for Lorentz factor $\gamma_0 = 4$. Here the impact parameter is $\rho = 6$ with $R_1 = 10, R_2 = 8, R_3 = 6$.

Figure 3.10: Trajectory of the electrons in transverse ellipsoid bubble at $C = -0.2$ and with bubble velocity $V = 0.969$ for Lorentz factor $\gamma_0 = 4$. Here the impact parameter is $\rho = 6$ with $R_1 = 10, R_2 = 8, R_3 = 6$.

Figure 3.11: Trajectory of the electrons in spherical bubble at $C = 0.0$ with bubble radius $R = 10$ and impact parameter $\rho = 10$. Here, different bubble velocities have been considered as $V_1 = 0.980, V_2 = 0.969, V_3 = 0.944$.

Figure 3.12: Trajectory of the electrons in longitudinal ellipsoid bubble at $C = 0.2$ with bubble radius $R = 10$ and with impact parameter $\rho = 10$. Here, different bubble velocities have been considered as $V_1 = 0.980, V_2 = 0.969, V_3 = 0.944$.

Figure 3.13: Trajectory of the electrons in transverse ellipsoid bubble at $C = -0.2$ with bubble radius $R = 10$ and impact parameter $\rho = 10$. Here, different bubble velocities have been considered as $V_1 = 0.980, V_2 = 0.969, V_3 = 0.944$.

Figure 4.1: Laser field E_y transverse to the propagation direction. Fig.4.1(a) represents the laser field for Gaussian laser pulse and fig.4.1 (b) represents the laser field for trapezoidal laser pulse at $t = 100 fs$.

Figure 4.2: Longitudinal electric field i.e. accelerating field E_x (TV/m) along the direction of beam propagation at time $t = 2.4 ps$. The solid line represents the electric field for a temporal Gaussian laser pulse and the dashed line represents the electric field for a temporal trapezoidal pulse.

Figure 4.3: Variation of longitudinal electric field E_x (TV/m) along (x, y) grid at time $t = 2.4 ps$ for temporal Gaussian laser pulse profile (a) and for temporal trapezoidal laser pulse profile (b).

Figure 4.4: Plasma electrons density for temporal Gaussian laser pulse driven LWFA in 4.4(a, c, e) and for temporal trapezoidal laser pulse driven LWFA in 4.4(b, d, f) at simulation time $t = 1.3 ps, 2.0 ps, 2.4 ps$.

Figure 4.5: Comparison of accelerated electrons kinetic energy spectra for Gaussian laser pulse (a) for trapezoidal laser pulse (b) at $t = 2.4 ps$.

Figure 4.6: Line spectra of accelerated electrons with their kinetic energy along x axis and corresponding electron number/GeV along y axis at time $t = 2.4 ps$. Solid line represents line spectra for Gaussian pulse and dashed line represents line spectra for trapezoidal pulse.

Figure 4.7: Variation of kinetic energy gradient in MJ/m with different time scale. Solid line represents kinetic energy gradient for Gaussian laser pulse driver as shown in figure 4.7(a) and dashed line represents kinetic energy gradient for trapezoidal laser pulse as shown in figure 4.7(b).

Figure 5.1: Illustration of laser profiles for a single pulse and two- and four-pulse trains.

Figure 5.2: Ion cavity behind the laser pulse, electron sheath and self-injected electrons at time $t = 1.6$ ps for single pulse.

Figure 5.3: Comparison of plasma density and electron bunch accelerated in the cavity at time $t = 4$ ps for (a) single laser pulse with normalised laser vector potential $a_0 = 5.2$ (b) two-pulse train with $a_0 = 3.68$ and (c) four-pulse train with $a_0 = 2.6$.

Figure 5.4: Comparison of longitudinal electric field (E_x) at time $t = 4$ ps for (a) single-pulse with normalised laser vector potential $a_0 = 5.2$ (b) two-pulse trains with $a_0 = 3.68$ and (c) four-pulse trains with $a_0 = 2.6$.

Figure 5.5: Poynting flux at time $t = 4$ ps for (a) single-pulse with normalised laser vector potential $a_0 = 5.2$ (b) two-pulse trains with $a_0 = 3.68$ and (c) four-pulse trains with $a_0 = 2.6$ (d) six-pulse trains with $a_0 = 2.1$ (e) eight-pulse trains with $a_0 = 1.831$.

Figure 5.6: The energy spectra of electrons accelerated in the wakefield of a single-pulse and train of multi-pulses corresponding to the laser parameters as given in table 1.

Figure 6.1: A trapezoidal plasma density profile with initial maximum density $n_e = 10^{19} \text{ cm}^{-3}$ with plasma region length 3.6 mm followed by a up ramp region, a plateau region and a down ramp region.

Figure 6.2: Longitudinal electric field E_x (TV/m) at time $t = 4.0$ ps along propagation direction. Three different curve represents electric field for three different up ramp length.

Figure 6.3: Electrons modulation with plot of charge density of plasma electrons ρ_e ($10^{20} \text{ cm}^{-3} e$) along propagation direction with (a) up ramp length 1.2 mm , down ramp length 1.2 mm and plateau length 1.2 mm (b) up ramp length 1.4 mm , down ramp length 1.2 mm and plateau length 1.0 mm and (c) up ramp length 1.6 mm , down ramp length 1.2 mm and plateau length 0.8 mm .

Figure 6.4: Energy spectrum of accelerated electrons with electron number /GeV along y axis with kinetic energy of accelerated electrons along x-axis at time $t = 4.0$ ps. Three scenarios have been shown in figure 6.4.

Figure 6.5: Kinetic energy gradient of accelerated electrons in (MJ/m) with different time scale and three curve represents with three different scenario of density profile. First one is with up ramp length 1.6 mm , down ramp length.

Figure 6.6: Plot of plasma electron density profile consisting of down-ramp and up-ramp.

Figure 6.7: (a) Plasma density modifications in the wake of laser pulse showing bubble and self-injected electrons at time $t = 3 \text{ ps}$ and (b) Creation of longitudinal electric field (wakefield) to accelerate the self-injected electrons.

Figure 6.8: Probe diagnostic of electron density (a) and longitudinal electric field (b) at time $t = 3 \text{ ps}$.

Figure 6.9: Energy spectrum of electrons showing electron acceleration in the wakefield at time $t = 3 \text{ ps}$ (a), and increase in kinetic energy gradient of the electrons as they are accelerated in the wakefield (b).

Figure 7.1: Mechanism of two ultrashort laser pulses and one is pump pulse and corresponding wakefield behind the pump pulse travelling from left side and second is injection pulse with lower laser pulse strength and corresponding wakefield behind the injection pulse traveling from right end.

Figure 7.2: Laser electric field E_y in y-direction for higher amplitude propagating from left of the boundary and for lower amplitude propagating from right to left at time $t=100 \text{ fs}$ for (a) GG system (b) GT system (c) TG system (d) TT system.

Figure 7.3: Longitudinal electric field E_x in x-direction after the simulation time $t = 100 \text{ ps}$ for (a) GG system (b) for GT system (c) for TG system and (d) for TT system.

Figure 7.4: Longitudinal electric field E_x in x-direction after the simulation time $t = 500 \text{ fs}$ for (a) GG system (b) for GT system (c) for TG system and (d) for TT system.

Figure 7.5: Longitudinal electric field E_x in x-direction after the simulation time $t = 2.4 \text{ ps}$ for (a) GG system (b) for GT system (c) for TG system and (d) for TT system.

Figure 7.6: Bubble formation in the plasma behind the probe pulse and injection pulse collision at $t=200 \text{ fs}$ for (a) GG system (b) GT system (c) TG system (d) TT system.

Figure 7.7: Bubble formation in the plasma behind the probe pulse and injection pulse collision at $t=500 \text{ fs}$ for (a) GG system (b) GT system (c) TG system (d) TT system.

Figure 7.8: Bubble formation in the plasma behind the probe pulse and injection pulse collision at $t=2.4 \text{ ps}$ for (a) GG system (b) GT system (c) TG system (d) TT system.

Figure 7.9: Energy spectrum of accelerated bunch of electrons at $t=2.4 \text{ ps}$ for GG system as shown in black curve, for GT system as shown in blue dashed curve, for TG system as shown in yellow dashed dotted curve and for TT system as shown in green dotted curve.

List of Symbols

a_0 :	Laser strength parameter
A :	Normalized vector potential
A_x :	Normalized vector potential along x-axis
A_y :	Normalized vector potential along y-axis
A_z :	Normalized vector potential along z-axis
A_{\perp} :	Normalized transverse vector potential
a :	Longitudinal geometrical coefficient associated with wakefield potential
b :	Transverse geometrical coefficient associated with wakefield potential
B :	Magnetic field (magnitude)
B_x :	Magnetic field along x-axis
B_y :	Magnetic field along y-axis
B_z :	Magnetic field along z-axis
c :	Speed of light in free space
C or C :	Geometrical coefficient associated with transverse vector potential
C_1, C_2, C_3, C_4, C_5 :	Different sets of Geometrical coefficient associated with transverse vector potential
E :	Electric field (magnitude)
E_0 :	Maximum electric field (magnitude)
E_x :	Electric field along x-axis
E_y :	Electric field along y-axis
E_z :	Electric field along z-axis
E_{pulse} :	Energy of the laser pulse
$-e$:	Charge of an electron
ϵ_0 :	Electrical permittivity in free space

η :	Geometrical parameter
F :	Restoring force according to Hooke's law
F_e :	Lorentz force
f :	Distribution function
H :	Hamiltonian equation
I :	Intensity of laser
I_0 :	Maximum intensity of laser
I_{18} :	Intensity in the units of 10^{18} W/cm^2
J :	Current density (magnitude)
J_x :	Current density along x-axis
J_{\perp} :	Transverse current density
k :	Spring constant
k_p :	Plasma wavevector
K :	Transverse geometrical coefficient
L_d :	Dephasing length
λ or λ_0 :	Laser's wavelength
λ_p :	Plasma wavelength
$\lambda_{\mu m}$:	Laser's wavelength in micrometer
m_e :	Mass of an electron
n :	number of laser pulse
n_e :	Initial plasma electron density
n_a :	Residual electron density
n_0 :	Ambient plasma electron density
n_c :	Critical density of plasma electrons
p :	Momentum of electron (magnitude)

p_x	: Longitudinal kinetic momentum of electrons
p_{x0}	: Initial kinetic momentum of electrons along x-axis
p_x	: Longitudinal kinetic momentum of electrons
p_{y0}	: Initial kinetic momentum of electrons along y-axis
p_y	: Transverse kinetic momentum of electrons
P_x	: Longitudinal canonical momentum of electrons
P_y	: Transverse canonical momentum of electrons
P_{peak}	: Peak power of laser system
P_c	: Threshold power of laser system
ϕ	: Scalar potential
Φ or Ψ	: Wakefield potential
r	: Three-dimensional position
r_b or R or R_b	: Radius of the bubble
R_1, R_2, R_3	: Different sets of the radius of the bubble
r_0	: Plasma length
ρ_e	: Charge density of electron
ρ	: Impact parameter
ρ_1, ρ_2, ρ_3	: Different sets of impact parameter
t	: Actual time
t_1	: Time at first position
t_2	: Time at second position
τ_G	: Time at HWHM
τ_L	: Laser's period
τ_p	: Time at FWHM
v	: Velocity of electrons

v_x :	Velocity of electrons along x-axis
v_g :	Group velocity of the laser pulse
v_{etch} :	Etching velocity of the photons
v_{ph} :	Phase velocity of the wakefield
v_b or V :	Bubble velocity
V_1, V_2, V_3 :	Different sets of bubble velocity
w_G :	Spot size at HWHM
w_0 :	Spot size at FWHM
ω_p :	Plasma frequency
ω or ω_0 or ω_L :	Laser's frequency
W_{max} or W_{gain} :	Energy gain
x :	Position along x-axis
x_1 :	Position along x-axis in first position
x_2 :	Position along x-axis in second position
γ :	Lorentz factor
γ_p :	Lorentz factor w.r.t. bubble velocity
γ_0 :	Initial Lorentz factor w.r.t. bubble velocity
y :	Position along y-axis
y_0 :	Initial position of impact parameter
ξ or ζ :	Normalized distance
ζ_0 :	Initial normalized distance
z :	Position along z-axis

List of Tables

Table 5.1: Laser parameters for one, two, four, six and eight pulses.

Table 5.2: Bubble radius and energy gain for one, two, four, six and eight pulses.

Table 7.1: Laser strength parameters for pump and injection pulses