

**DATA ENVELOPMENT ANALYSIS MODELS FOR
EFFICIENCY EVALUATION AND PRODUCTIVITY
ASSESSMENT IN THE BACKDROP OF DISPARATE DATA
AND STRUCTURES**

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by

Pooja Bansal

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Submitted

*in fulfillment of the requirements of the degree of Doctor of Philosophy
to the*



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APRIL 2021

Dedicated To
My Family

Certificate

This is to certify that the thesis entitled **Data Envelopment Analysis Models For Efficiency Evaluation and Productivity Assessment in the Backdrop of Disparate Data and Structures** submitted by **Ms. Pooja Bansal** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bonafide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

While studying the efficiency analysis, the first question that strikes one's mind is why we need to bother about units' relative efficiency. The answer to this question lies in the fact that the efficiency assessment is vital and invigorating. In today's competitive world, efficiency evaluation contributes to procuring the target, performance improvement, and promoting desired abilities. The breakthrough research study by Charnes, Cooper, and Rhodes in 1978, which presented the application of linear programming to approximate the production technology frontier, has been the gist for the introduction of Data Envelopment Analysis (DEA) as a tool for measuring efficiency and productivity of decision-making units (DMUs). DEA's worth lies in its accomplishment to relatively measuring the efficiency of a DMU within a target group of interest that operates in specific application realms, both public and private sectors, such as the banking industry, health care industry, education sector, and supply chain management, etc. There has been exponential growth in the number of publications related to DEA theory and applications in recent years. A desire for enhancing DEA by reducing its disadvantages or strengthening its advantages has been a significant cause of many recent literature discoveries.

This thesis aims to fabricate novel theoretical DEA models to bargain with a few limitations on data in different DEA frameworks. The arrangement is carried forward in two dimensions: efficiency assessment and productivity change measurement.

We design a multi-period DEA model considering desirable and undesirable input-output features with negative and non-negative data values. The proposed model possesses the requisite features of translation invariance and unit independence, obligatory when dealing with the negative data set. Furthermore, in the same work, we introduced a super-efficiency model in a multi-period DEA framework to further discriminate the performance of efficient DMUs.

In many real-life circumstances, the input and output data features may expect only integer values. In certain situations, rounding of the efficiency measurement to the closest whole number could be deluding. We utilize the directional distance function (DDF) approach for tackling DEA problems with positive and non-positive integer and continuous data. To count the impact of carryovers connecting consecutive periods, we introduce an integer dynamic DEA model to measure units' efficiency and super-efficiency over a specific period. We design the direction

vectors so that the issue of infeasibility often encountered in super-efficiency DEA models is resolved entirely.

In another research study in this thesis, a new integrated dynamic interval DEA model is proposed to perform the efficiency analysis of DMUs to accommodate interval-valued and integer-valued features that can take negative values. The proposed one-step model follows the DDF approach to determine the efficiency of DMUs involving the carryovers between the consecutive periods. We use the pessimistic and optimistic standpoints to estimate the respective lower and upper bounds of the interval efficiency scores of the DMUs.

Besides modeling for efficiency assessment of DMUs, this thesis introduces a framework to measure productivity change in the DMUs in the presence of negative data over two consecutive periods. In our findings, we detect the infeasibility of the DDF based DEA model of Malmquist-Luenberger productivity index (MLPI) under the variable returns to scale (VRS) technology when the data takes on negative values. We address this issue and formulate a new DDF based DEA model to compute an improved MLPI. This research work's gist lies in consolidating carryover variables connecting successive periods and intermediate products relating divisions simultaneously. More specifically, we work with the dynamic network DEA framework to introduce the dynamic MLPI and dynamic sequential MLPI (DSMLPI) to capture the shifts in the efficient frontiers induced by random shocks or innovations in technology over the period. Furthermore, we introduce an interval DEA model to measure the productivity change of DMUs applying the SMLPI approach. Moreover, to capture the environmentally sensitive productivity growth of DMUs operating under heterogeneous production technologies, we develop a dynamic metafrontier MLPI (DMMLPI). The DMMLPI offers a tripartite decomposition of the productivity change index into three distinct drivers: efficiency change (EC), best-practice change (BPC), and technology gap change (TGC).

The core of the thesis lies in its acquaintance of theoretical models to deal with certain restrictions on data in different DEA frameworks. The methodologies introduced and developed in this thesis are applied to the real data sets of various sectors like the Indian banking sector, airlines, Indian institutes of technologies Institutes, and economic data of the some countries.

सार

दक्षता विश्लेषण का अध्ययन करते समय, पहला सवाल जो किसी के दिमाग पर हमला करता है, हमें इकाइयों की सापेक्ष दक्षता के बारे में परेशान होने की आवश्यकता क्यों है। इस सवाल का जवाब इस तथ्य में निहित है कि दक्षता मूल्यांकन महत्वपूर्ण और स्फूर्तिदायक है। आज की प्रतिस्पर्धी दुनिया में, दक्षता मूल्यांकन लक्ष्य की खरीद, प्रदर्शन में सुधार और वांछित क्षमताओं को बढ़ावा देने में योगदान देता है। 1978 में चर्न्स, कूपर और रोड्स द्वारा किया गया सफलता शोध अध्ययन, जिसने उत्पादन तकनीक की सीमा को अनुमानित करने के लिए लीनियर प्रोग्रामिंग के अनुप्रयोग को प्रस्तुत किया, जो डेटा एनवलपमेंट एनालिसिस (DEA) को निर्णय लेने वाली इकाइयों (DMUs) की दक्षता और उत्पादकता को मापने के लिए एक उपकरण के रूप में प्रस्तुत करने के लिए किया गया है। DEA का मूल्य ब्याज की एक लक्षित समूह के भीतर एक DMU की दक्षता को मापने के लिए उसकी उपलब्धि में निहित है, जो विशिष्ट अनुप्रयोग क्षेत्र में, सार्वजनिक और निजी दोनों क्षेत्रों में संचालित होता है, जैसे कि बैंकिंग उद्योग, स्वास्थ्य देखभाल उद्योग, शिक्षा क्षेत्र और आपूर्ति श्रृंखला प्रबंधन, आदि। हाल के वर्षों में DEA सिद्धांत और अनुप्रयोगों से संबंधित प्रकाशनों की संख्या में घातीय वृद्धि हुई है। इसके नुकसान को कम करके या इसके फायदे को मजबूत करके DEA बढ़ाने की इच्छा हाल ही की कई साहित्य खोजों का महत्वपूर्ण कारण रही है।

इस थीसिस का उद्देश्य विभिन्न DEA ढांचे में डेटा पर कुछ सीमाओं के साथ उपन्यास सैद्धांतिक DEA मॉडल तैयार करना है। व्यवस्था को दो आयामों में आगे बढ़ाया जाता है: दक्षता मूल्यांकन और उत्पादकता परिवर्तन माप।

हम नकारात्मक और गैर-नकारात्मक डेटा मानों के साथ वांछनीय और अवांछनीय इनपुट-आउटपुट सुविधाओं पर विचार करते हुए एक बहु-अवधि DEA मॉडल डिजाइन करते हैं। प्रस्तावित मॉडल में अपेक्षित विशेषताएं हैं, अनुवाद निश्चरता और इकाई स्वतंत्रता, जो नकारात्मक डेटा सेट के साथ काम करने के लिए अनिवार्य है। इसके अलावा, एक ही काम में, हमने कुशल DMU के प्रदर्शन में भेदभाव करने के लिए एक बहु-अवधि DEA ढांचे में एक सुपर-दक्षता मॉडल पेश किया।

कई वास्तविक जीवन की परिस्थितियों में, इनपुट और आउटपुट डेटा सुविधाएँ केवल पूर्णांक मानों की अपेक्षा कर सकती हैं। कुछ स्थितियों में, निकटतम पूर्ण संख्या के लिए दक्षता माप की गोलाई बहक सकती है। हम सकारात्मक और गैर-सकारात्मक पूर्णांक और निरंतर डेटा के साथ DEA समस्याओं से निपटने के लिए दिशात्मक दूरी फ़ंक्शन (DDF) दृष्टिकोण का उपयोग करते हैं। एक विशिष्ट अवधि में लगातार अवधि को जोड़ने वाले कैरीओवर के प्रभाव को गिनने के लिए, हम यूनिट्स की दक्षता और सुपर-दक्षता को मापने के लिए एक पूर्णांक डायनेमिक DEA मॉडल पेश करते हैं। हम दिशा वैक्टर को डिजाइन करते हैं ताकि सुपर-दक्षता DEA मॉडल में अक्सर होने वाली घुसपैठ की समस्या पूरी तरह से हल हो जाए।

इस थीसिस में एक अन्य शोध अध्ययन में, एक नए एकीकृत गतिशील अंतराल DEA मॉडल को DMU की दक्षता विश्लेषण करने के लिए प्रस्तावित किया गया है ताकि अंतराल-मूल्य और पूर्णांक-मूल्यवान विशेषताओं को समायोजित किया जा सके जो नकारात्मक मान ले सकें। प्रस्तावित एक-चरण मॉडल लगातार अवधि के बीच कैरीओवर को शामिल करने वाले DMUs की दक्षता निर्धारित करने के लिए DDF दृष्टिकोण का अनुसरण करता है। हम DMUs के अंतराल दक्षता स्कोर के संबंधित निचले और ऊपरी सीमा का अनुमान लगाने के लिए निराशावादी और आशावादी दृष्टिकोण का उपयोग करते हैं।

DMU के दक्षता मूल्यांकन के लिए मॉडलिंग के अलावा, यह थीसिस लगातार दो अवधियों में नकारात्मक डेटा की उपस्थिति में DMU में उत्पादकता परिवर्तन को मापने के लिए एक रूपरेखा प्रस्तुत करती है। हमारे निष्कर्षों में, जब डेटा नकारात्मक मानों पर ले जाता है तो हम वैरिएबल रिटर्न टू स्केल (VRS) तकनीक के तहत मैल्मक्विस्ट-लुएंबर्गर उत्पादकता सूचकांक (MLPI) के DDF आधारित DEA मॉडल की घुसपैठ का पता लगाते हैं। हम इस समस्या का समाधान करते हैं और एक बेहतर MLPI की गणना करने के लिये नया DDF आधारित DEA मॉडल तैयार करते हैं। इस अनुसंधान कार्य का सार एक साथ विभाजन से संबंधित मध्यवर्ती उत्पादों और समय को जोड़ने वाले कैरीओवर चर को समेकित करने में निहित है। अधिक विशेष रूप से, हम गतिशील नेटवर्क DEA फ्रेमवर्क के साथ काम करते हैं, जो कि गतिशील MLPI (DMLPI) और गतिशील अनुक्रमिक MLPI (DSMLPI) को पेश करते हैं, जो कि अवधि में प्रौद्योगिकी के यादृच्छिक झटकों या नवाचारों से प्रेरित कुशल सीमाओं में बदलाव को पकड़ने के लिए है। इसके अलावा, हम SMLPI दृष्टिकोण को लागू करने वाले DMUs के उत्पादकता परिवर्तन को मापने के लिए एक अंतराल DEA मॉडल पेश करते हैं। इसके अलावा, विषम उत्पादन तकनीकों के तहत काम कर रहे DMU के पर्यावरण की दृष्टि से संवेदनशील उत्पादकता वृद्धि को पकड़ने के लिए, हम एक गतिशील मेटाफॉर्स्टियर MLPI (DMMLPI) विकसित करते हैं। DMMLPI तीन अलग-अलग ड्राइवर्स में दक्षता परिवर्तन सूचकांक के एक त्रिपक्षीय अपघटन प्रदान करता है: दक्षता परिवर्तन (EC), सर्वोत्तम-अभ्यास परिवर्तन (BPC), और प्रौद्योगिकी अंतर परिवर्तन (TGC)।

थीसिस का मूल विभिन्न DEA ढांचे में डेटा पर कुछ प्रतिबंधों से निपटने के लिए सैद्धांतिक मॉडल के अपने परिचित में निहित है। इस थीसिस में पेश और विकसित की गई पद्धतियाँ विभिन्न क्षेत्रों जैसे भारतीय बैंकिंग क्षेत्र, एयरलाइंस, भारतीय संस्थानों के संस्थानों और कुछ देशों के आर्थिक आंकड़ों के वास्तविक डेटा सेटों पर लागू होती हैं।

Contents

Certificate	i
Acknowledgements	iii
Abstract	v
	vii
List of Figures	xiii
List of Tables	xvii
List of Abbreviations	xxi
1 Introduction and Preliminaries	1
1.1 Introduction	1
1.2 Overview of DEA	4
1.2.1 Radial DEA models	5
1.2.2 Non-Radial DEA Models	11
1.2.3 DEA Models with Negative Data	15
1.2.4 Super-efficiency DEA Models	17
1.2.5 DEA Models with Undesirable Data	21
1.2.6 DEA Models with Integer Data	23
1.2.7 Interval DEA Models	25
1.2.8 Dynamic DEA	28
1.2.9 Network DEA	31
1.3 Productivity Change Measurement Over Time	31
1.3.1 Malmquist Productivity Index (MPI)	32
1.3.2 Luenberger Productivity Index (LPI)	34
1.3.3 Malmquist-Luenberger Productivity Index (MLPI)	35

1.3.4	Sequential Productivity Indices	37
1.3.5	Metafrontier Productivity Indices	39
1.4	Organization of the Thesis	40
2	Multi-period Super-efficiency DEA Models	45
2.1	Motivation and Objective	45
2.2	Multi-period Technical, Ecological, and Overall Efficiency	47
2.3	A Multi-period Additive Model	49
2.4	A Multi-period Super-Efficiency Model	52
2.5	Illustration	57
2.6	Dynamic Additive DEA Models	60
2.7	Conclusions	64
3	Super Efficiency In Integer Dynamic DEA	67
3.1	Motivation and Objective	67
3.2	Modified DDF based Integer DEA Model for Efficiency Evaluation	68
3.3	DDF based Integer Super-efficiency DEA Model	71
3.4	DDF based Integer Dynamic DEA Models	76
3.5	Numerical Illustration	80
3.6	Conclusions	83
4	Integrated Interval DEA Model with Integer Data	85
4.1	Motivation and Objective	85
4.2	Dynamic Integrated Interval DEA Model	86
4.3	Classification and Ranking of DMUs	96
4.4	Comparison Analysis	96
4.5	Empirical Illustration	104
4.6	Conclusions	106
5	Malmquist-Luenberger Productivity Indexes in Dynamic Network Framework	107
5.1	Motivation and Objective	107
5.2	Overview of MLPI and Infeasibility Concern	108
5.3	Dynamic Network DEA Structure	114
5.4	Dynamic ML Productivity Index	116
5.5	Dynamic Sequential ML Productivity Index	125
5.6	Sequential Window analysis	130
5.7	Conclusions	132

Contents	xi
6 An Illustrated Application for Assessing Productivity Performance of Indian Banks	133
6.1 Introduction	133
6.1.1 Data	136
6.1.2 Results and discussion	137
6.2 DSMLPI using sequential window analysis	142
6.3 Conclusions	145
7 Dynamic Metafrontier Malmquist-Luenberger Productivity Index in Network DEA	147
7.1 Motivation and Objective	147
7.2 Methodology	148
7.2.1 Structure	148
7.2.2 Production Technologies	149
7.2.3 DDF Models	151
7.3 Empirical Illustration	155
7.3.1 Data	155
7.3.2 Results	156
7.4 Conclusions	161
8 Sequential Malmquist-Luenberger Productivity Index with Interval data	163
8.1 Motivation and Objective	163
8.2 Methodology	164
8.2.1 Interval SMLPI under CRS Technology	164
8.2.2 Interval SMLPI under VRS Technology	168
8.3 Empirical Illustration	172
8.3.1 Data	173
8.3.2 Results and Discussion	173
8.4 Conclusions	174
9 Contributions, Limitations and Future Outlook	177
9.1 Contributions of the Thesis	177
9.2 Limitations and Future Outlook	178
References	181
Curriculum Vitae	197

List of Figures

1.1	Representation of data in one input and two output case.	2
1.2	Various DEA Models and productivity change measurement indexes utilizing DEA approach.	5
1.3	Input oriented CCR (in red) and BCC (in blue) efficient frontiers	9
1.4	super-efficiency under VRS postulate by Chen et al. [29]	19
1.5	Effects of rounding up and rounding down the target for single input and two output case.	24
1.6	The dynamic framework of T periods for a DMU_o ; the consecutive periods are connected through desirable and undesirable carryovers, the carryovers at the final period T are considered as outputs.	30
1.7	The basic network series structure with K divisions.	31
1.8	The network series structure with K divisions with each division consuming exogenous inputs.	32
1.9	The basic parallel network structure with K divisions.	32
1.10	The distance measures computed for the output oriented MPI for DMUs A^t and A^{t+1} utilizing the data of period t and $t + 1$, are $\tilde{A}^t$, $\hat{A}^t$, $\tilde{A}^{t+1}$, and $\hat{A}^{t+1}$, respectively. The figure is inspired from section 8.3 of [94].	34
1.11	The directional distance measures computed for the LPI for DMUs A^t and A^{t+1} utilizing the data of period t and $t + 1$, are $\tilde{A}^t$, $\hat{A}^t$, $\tilde{A}^{t+1}$, and $\hat{A}^{t+1}$, respectively. The figure is inspired from section 8.4 of [94].	36
1.12	Graphical illustration of group frontier and metafrontier production technologies.	40
2.1	weight function w^t for different values of ξ in $[0, 1]$ at $t = 1, \dots, 5$	53
2.2	Multi-period and dynamic architecture in three periods $t - 1$, t , and $t + 1$	60
2.3	Input-output and carryovers of Iranian banks for three consecutive period.	63

3.1	The efficient frontier of 10 DMUs with one input and one output; Figures (3.1a) and (3.1b) describe the efficient frontier (in blue line) and direction vectors (in orange lines) for the inefficient DMUs to achieve the efficient frontier from model (M3.1) under integer constraint on the variables and without integer restriction, respectively, for $\ell = 1, k = 3$; points in green on the efficient frontier in figure (3.1b) depict the real targets of the inefficient DMUs.	74
3.2	The efficient frontier of 10 DMUs with one input and one output; Figures (3.2a) and (3.2b) describe the efficient (in blue line) and direction vectors (in orange lines) for the inefficient DMUs to achieve the efficient frontier from model (M3.1) by imposing integer constraint on two variables and without integer restriction, respectively, for $\ell = 10, k = 30$; points in green on the efficient frontier in figure (3.2b) show the real targets for the inefficient DMUs.	75
3.3	The directions for efficient DMUs to obtain the super efficiency frontier either by increasing their inputs or reducing their outputs when the parameters are set at $\ell' = 1, k' = 3$; the red dotted lines depict the efficient frontiers obtained by removing the respective DMUs, and the orange lines show the directions for efficient DMUs to achieve this new efficient frontier.	75
3.4	The dynamic structure of an Indian Institute of technology (IITs) in two consecutive periods.	81
4.1	The dynamic framework for a DMU comprising of T periods with multiple carryovers connecting consecutive periods.	87
4.2	The efficient frontiers of two super-efficient units for data in Table 4.1. (a) and (b) describe the upper and lower bounds of efficient frontiers for DMUs 1 (red) and 3 (blue).	95
4.3	The dynamic structure of an airline in two consecutive periods.	104
5.1	Projections of DMUs data at $t + 1$ to the efficient frontier formed by the data of the DMUs at period t . The efficient frontier at t is depicted in blue color, the green dots represent the data of DMUs at $t + 1$, and the projections of $t + 1$ data on the efficient frontier at t along the proposed direction vectors are in red color.	114
5.2	The dynamic network series structure with K divisions and T periods of analysis. The divisions are connected vertically through intermediate links and horizontally across time by the carryovers.	115

5.3	The dynamic network structure of a DMU with 2 divisions and 3 periods of analysis.	122
5.4	The technical change in DMLPI of 30 DMUs in two consecutive periods. .	126
5.5	The technical change in DSMLPI in two consecutive periods $t = 1 \& 2$ and $t = 2 \& 3$ on the synthetic data of 30 DMUs.	130
6.1	The three-stage dynamic network structure of a public sector bank.	134
6.2	Efficiency change, technical change, and productivity change indices for Indian public sector banking industry.	140
6.3	DSMLPI and its components efficiency change and technical change for Indian public banks over the period of 2005-2017 using three-year window analysis approach.	144
7.1	The classification of DMUs into G groups where the DMUs follow homogeneous (within the group) and heterogeneous (across the groups) production technologies.	148
7.2	Graphical depiction of contemporaneous, intertemporal, and metafrontier production technologies.	151
7.2	Trends in the DMMLPI and its decomposition. (a) Efficiency change (EC) (b) Best practice change (BPC) (c) Technological gap change (TGC) (d) DMMLPI.	160
7.3	Group-wise average decomposition factors of DMMLPI.	161
8.1	The structure of an Indian commercial bank.	173

List of Tables

1.1	Data of eight DMUs with one input and two output.	2
1.2	Input-output data of six DMUs and the results of CCR and BCC models. .	9
2.1	Data including undesirable outputs and non-positive data values.	51
2.2	Results from (M2.4) model and the MSBM model (M1.11) on the data in Table 2.1.	51
2.3	Results from models (M2.4) and (M2.5) ($\xi = 0.7$ for model (M2.5)) and models numbered (7) and (8) in [91] on the data reported in [91]. Here, σ^* and ρ^* are super-efficiency scores of efficient DMUs by model (M2.5) and model numbered (8) in [91], respectively. Also, the symbol ‘-’ in columns 4 and 8 indicates that the DMU is inefficient and not a candidate for super-efficiency model (M2.5).	56
2.4	Results from models (M2.4) and (M2.5) ($\xi = 0.7$ for model (M2.5)) and models in [58] on the data reported in [58]. Here σ^* is the super-efficiency score of efficient DMUs by model (M2.5) and ψ_o^* is the optimal value of the model in [58]. Also, the symbol ‘-’ in columns 5 indicates that the DMU is inefficient and not a candidate for super-efficiency model (M2.5).	57
2.5	Results of models (M2.4) and (M2.5) on a multi-period data of 37 countries where $\xi = 0.7$ in model (M2.5). The symbol ‘-’ in column 5 corresponds to inefficient DMU which is not a candidate for super-efficiency model (M2.5).	59
2.6	Data set of 10 banks reported in [163] with two inputs, one output, one desirable and one undesirable carryover for three periods.	64
2.7	Results of models (M2.7) and (M2.8) ($\xi = 0.7$ for model (M2.8)) by including and excluding desirable and undesirable carryovers on the data reported in Table 2.6. Here, the symbol ‘-’ in columns 5 and 9 indicate that the DMU is inefficient and not a candidate of super-efficiency model (M2.8).	64

3.1	Results from models (M3.1) and (M3.2) when $\ell = \ell' = 1$ and $k = k' = 3$, when (i) input-output features are restricted to take integer values, (ii) the integer constraint is relaxed to allow both features to take real values. Also, the symbol ‘-’ in columns 7 and 11 indicate that the DMU is inefficient and not a candidate for super-efficiency model (M3.2).	74
3.2	Efficiency and super-efficiency analysis on an illustrative data set spill over three time periods by applying dynamic models (M3.3) and (M3.4) setting the parameters $\ell = \ell_1 = \ell'_1 = 1$ and $k = k' = k_1 = 3$	80
3.3	Inputs, outputs and carryovers used in the efficiency analysis on the research performance of 13 IITs.	81
3.4	Results from dynamic models (M3.3) and (M3.4), and the multi-period DEA model applied on the data set of various IITs where ψ^* is the arithmetic average of inefficiency scores in a particular period under study. The symbol ‘-’ in column 3 indicates that the DMU is inefficient and not a candidate for super-efficiency model (M3.4).	82
4.1	Results from models (M4.1) and (M4.2) on the data of five DMUs when $\ell = \ell' = 1$ and $k = k' = 3$; the output is restricted to take only integer values in the intervals.	95
4.2	Results from models (M4.1) and (M4.2), $\ell = \ell' = 1$, $k = k' = 3$, in columns 2 and 3; and the interval SORM model [87] in columns 5 and 6 for the data set of 20 banks described in [87]. Column 4 represents the classification of DMUs into three classes by our approach, while column 7 presents the classification into strictly efficient class (E^{++}), weakly efficient class (E^+), and inefficient class (E^-) defined in [87].	97
4.3	Results from models (M4.1) and (M4.2) with necessary modifications, and the InDEA model in [107] on the data described in [107]. Here, E^{Lt} and E^{Ut} denote the lower and the upper bounds of the efficiency interval obtained by our models at time t , while these bounds are represented by q^{Lt} and q^{Ut} , respectively, for the models in [107]. The symbols Q , Q^+ and Q^{++} indicate the inefficient, efficient but not super-efficient, and super-efficient DMUs, respectively, by the classification defined in [107]. The ranking of units within each class is carried out by the PD method and recorded for both our proposed models and the one in [107].	99
4.4	Synthetic data set of 30 DMUs for $t = 1$ with two inputs, two outputs, and one desirable carryover connecting the two periods.	101
4.5	Synthetic data set of 30 DMUs for $t = 2$ with two inputs, two outputs. . .	102

4.6	Result from models (M4.1) and (M4.2) with $\ell = \ell' = 1$ and $k = k' = 3$, on the two period synthetic data set of 30 DMUs given in Tables 4.4 and 4.5.	103
4.7	Inputs, outputs, desirable and undesirable carryovers applied in the empirical analysis.	104
4.8	Data of 11 Indian airlines for the period 2014-15.	105
4.9	Results of the dynamic integrated interval efficiency models (M4.1) and (M4.2) with $\ell = \ell' = 1$ and $k = k' = 3$, and ranking of super-efficient and efficient DMUs.	106
5.1	The two period data and cross-period efficiency scores by model (M5.1).	110
5.2	The cross-period efficiency by model (M5.2), with $\pi = 1$, $\xi = 3$, $\xi' = 3$, on the data in Table 5.1.	113
5.3	Projections of $t + 1$ period input-output data to the t^{th} period efficient frontier, where $D^t(X^{t+1}, Y^{t+1})$ is computed by model (M5.2) with $\pi = 1$ and $\xi = 3$.	113
5.4	Synthetic data set 30 DMUs for period $t = 1$, and divisions $k = 1, 2$.	123
5.5	Synthetic data set of 30 DMUs for period $t = 2, 3$, and divisions $k = 1, 2$.	124
5.6	The DMLPI and its components for 30 DMUs on the synthetic data setting parameters $\xi = \xi' = \xi'' = \xi''' = 3$, and $\pi = \pi' = \pi'' = 1$.	125
5.7	The DSMLPI and its components on synthetic dataset in Tables 5.4 and 5.5 setting parameters $\eta_1 = \eta'_1 = \eta''_1 = \eta'''_1 = 3$, $\delta_1 = \delta'_1 = \delta''_1 = 1$ for period t , and $\eta_1 = \eta'_1 = \eta''_1 = \eta'''_1 = 1.5$, $\delta_1 = \delta'_1 = \delta''_1 = 1$ for period $t + 1$; $t = 1, 2$.	129
6.1	The inputs, outputs, intermediates, and carryovers for period t employed in the empirical analysis.	134
6.2	Summary statistics of variables used in the empirical illustration; the unit of measurement of all variables (except labor) is INR lakhs.	137
6.3	Year-wise estimates of DMLPI, DSMLPI, and its components during the sample period.	138
6.4	Club convergence results	142
6.5	Average efficiency, technical and productivity changes in public sector banks over a period of 2005-2017.	143
6.6	The average DMLPI and DSMLPI productivity indices and its components for public sector banks.	144
7.1	Summary statistics of variables used in the empirical illustration. The unit of measurement of all variables (except labor) is INR lakhs.	157

7.2	Average values of efficiency change (EC), best practice change (BPC), technical gap change (TGC), and DMMLPI in 2013-2017.	158
7.3	Geometric average of the three change factors for all the private, public and foreign banks for the period 2013-2017; the best value in the column is in bold.	160
8.1	The inputs, desirable and undesirable outputs employed in the empirical analysis.	172
8.2	Normalized data of 2018 for 21 Indian commercial banks; except labour, the bounds of the data values are measured in lakhs.	174
8.3	Productivity change intervals applying models (M8.2) and (M8.3).	175
8.4	Scale efficiency change intervals for Indian commercial banks from 2011 to 2018.	175

List of Abbreviations

DEA	Data envelopment analysis
DMU	Decision-making unit
PPS	Production possibility set
RTS	Returns to scale
CRS	Constant returns to scale
VRS	Variable returns to scale
TE	Technical efficiency
ECE	Ecological efficiency
OE	Overall efficiency
SBM	Slack based measure
DDF	Directional distance function
MSBM	Modified slack based measure
InDEA	Interval data envelopment analysis
MPI	Malmquist productivity index
LPI	Luenberger productivity index
MLPI	Malmquist-Luenberger productivity index
SMLPI	Sequential Malmquist-Luenberger productivity index
MMLPI	Metafrontier Malmquist-Luenberger productivity index
DMLPI	Dynamic Malmquist-Luenberger productivity index
DSMLPI	Dynamic Sequential Malmquist-Luenberger productivity index
DMMLPI	Dynamic Metafrontier Malmquist-Luenberger productivity index