

WIND INDUCED VIBRATION OF TALL CHIMNEYS

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WIND INDUCED VIBRATION OF TALL CHIMNEYS

by

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Submitted

*In partial fulfillment of the requirements of the degree of Doctor
of Philosophy to the*



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MAY 2024

Dedicated to
My family.

CERTIFICATE

This is to certify that the thesis entitled “**WIND INDUCED VIBRATION OF TALL CHIMNEYS**”, submitted by **Ms Saba Rahman** to the Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy in Department of Civil Engineering, is a record of the original, bona fide research work carried out by her under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

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Saba Rahman

ABSTRACT

Chimneys, being tall and slender structures, are particularly vulnerable to wind forces. Wind induced vibrations can lead to structural damage, increased maintenance costs, and potential safety issues. To ensure the safety and long-term performance of chimneys, assessing their dynamic reaction to wind loads and developing effective mitigating strategies is essential. This thesis offers a study of the wind induced vibration of chimneys using analytical and numerical models. Mitigating approaches used to suppress undesirable dynamic responses have also been developed.

Vortex induced vibration is a subset of wind induced vibration, which is vital to wind-prone structures as it is the primary source of development of potentially dangerous across-wind vibrations, making it crucial to understand and explore this phenomenon. Several approaches are available for assessing across-wind vibrations caused by vortex induced vibrations. Although, many of these approaches linearize aeroelasticity in their proposed analytical solution of the response of chimneys. Thus, the authors have proposed modified methods such as Direct spectral approach, Modal method, Wake oscillator model which duly considers nonlinearity into the analysis to obtain realistic response of chimneys. These models use analytical expressions and numerical simulations to predict the behavior of chimneys under vortex induced vibrations. Using these models, engineers or researchers can thoroughly study the impacts of vortex shedding on concrete chimneys and estimate its potential consequences. Furthermore, reliability analysis is also presented to report the uncertainty of the parameters impacting the behavior of concrete chimneys submitted to vortex induced forces. Most international wind codes and standards employ the deterministic and Vickery and Basu methods to offer their provisions. But the basis of the formulations of codes and standards is not much understood among wind practitioners. Therefore, this research presents a comparative examination of the major international wind codes to understand the formulation of code provisions.

To mitigate the response of the chimney subjected to vortex induced vibration, the authors have proposed techniques of passive dampers. One of the most common dampers is the tuned mass damper (TMD) and with technological advancements, inerter based dampers such tuned mass

inertor system (TMIS) have gain popularity. Although application of TMIS to the chimney to suppress vortex shedding has not been explored. Thus, the thesis focuses on the effectiveness of various passive dampers, such as tuned mass damper (TMD), tuned mass inertor system (TMIS) and multi-tuned mass damper (MTMD), on the dynamic response of concrete chimneys.

सार

चिमनी, लंबी और पतली संरचना होने के कारण, विशेष रूप से पवन बलों के प्रति संवेदनशील होती हैं। हवा से प्रेरित कंपन से संरचनात्मक क्षति, रखरखाव लागत में वृद्धि और संभावित सुरक्षा समस्याएं हो सकती हैं। चिमनियों की सुरक्षा और दीर्घकालिक प्रदर्शन सुनिश्चित करने के लिए, हवा के भार के प्रति उनकी गतिशील प्रतिक्रिया का आकलन करना और प्रभावी शमन रणनीति विकसित करना आवश्यक है। यह थीसिस विश्लेषणात्मक और संख्यात्मक मॉडल का उपयोग करके चिमनी के हवा से प्रेरित कंपन का अध्ययन प्रस्तुत करती है। अवांछनीय गतिशील प्रतिक्रियाओं को दबाने के लिए उपयोग किए जाने वाले शमन दृष्टिकोण भी विकसित किए गए हैं।

भंवर प्रेरित कंपन पवन प्रेरित कंपन का एक उपसमूह है, जो पवन-प्रवण संरचनाओं के लिए महत्वपूर्ण है क्योंकि यह संभावित रूप से खतरनाक पार-पवन कंपन के विकास का प्राथमिक स्रोत है, जिससे इस घटना को समझना और पता लगाना महत्वपूर्ण हो जाता है। भंवर प्रेरित कंपन के कारण होने वाले हवा के कंपन का आकलन करने के लिए कई दृष्टिकोण उपलब्ध हैं। हालाँकि, इनमें से कई दृष्टिकोण चिमनी की प्रतिक्रिया के अपने प्रस्तावित विश्लेषणात्मक समाधान में एयरोइलास्टिकिटी को रैखिक बनाते हैं। इस प्रकार, लेखकों ने प्रत्यक्ष वर्णक्रमीय दृष्टिकोण, मोडल विधि, वेक ऑसिलेटर मॉडल जैसे संशोधित तरीकों का प्रस्ताव दिया है जो चिमनी की यथार्थवादी प्रतिक्रिया प्राप्त करने के लिए विश्लेषण में गैर-रैखिकता पर विचार करता है। ये मॉडल भंवर प्रेरित कंपन के तहत चिमनी के व्यवहार की भविष्यवाणी करने के लिए विश्लेषणात्मक अभिव्यक्तियों और संख्यात्मक सिमुलेशन का उपयोग करते हैं। इन मॉडलों का उपयोग करके, इंजीनियर या शोधकर्ता कंक्रीट चिमनी पर भंवर बहाव के प्रभावों का गहन अध्ययन कर सकते हैं और इसके संभावित परिणामों का अनुमान लगा सकते हैं। इसके अलावा, भंवर प्रेरित बलों को प्रस्तुत कंक्रीट चिमनी के व्यवहार

को प्रभावित करने वाले मापदंडों की अनिश्चितता की रिपोर्ट करने के लिए विश्वसनीयता विश्लेषण भी प्रस्तुत किया गया है। अधिकांश अंतर्राष्ट्रीय पवन कोड और मानक अपने प्रावधानों की पेशकश के लिए नियतात्मक और विकरी और बसु विधियों का उपयोग करते हैं। लेकिन पवन विशेषज्ञों के बीच कोड और मानकों के निर्माण के आधार को ज्यादा नहीं समझा गया है। इसलिए, यह शोध कोड प्रावधानों के निर्माण को समझने के लिए प्रमुख अंतरराष्ट्रीय पवन कोड की तुलनात्मक परीक्षा प्रस्तुत करता है।

भंवर प्रेरित कंपन के अधीन चिमनी की प्रतिक्रिया को कम करने के लिए, लेखकों ने निष्क्रिय डैम्पर्स की तकनीक प्रस्तावित की है। सबसे आम डैम्पर्स में से एक ट्यून्ड मास डैम्पर (टीएमडी) है और तकनीकी प्रगति के साथ, इनरटर आधारित डैम्पर्स जैसे ट्यून्ड मास इनरटर सिस्टम (टीएमआईएस) ने लोकप्रियता हासिल की है। हालाँकि भंवर बहाव को दबाने के लिए चिमनी में टीएमआईएस के अनुप्रयोग का पता नहीं लगाया गया है। इस प्रकार, थीसिस कंक्रीट चिमनी की गतिशील प्रतिक्रिया पर विभिन्न निष्क्रिय डैम्पर्स, जैसे ट्यून्ड मास डैम्पर (टीएमडी), ट्यून्ड मास इनरटर सिस्टम (टीएमआईएस) और मल्टी-ट्यून्ड मास डैम्पर (एमटीएमडी) की प्रभावशीलता पर केंद्रित है।

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List of Abbreviations

A, ε	Van der Pol parameters
b	Mass of inertance
B_w	Bandwidth of vortex shedding
c	Combined damping of structure and fluid
c_{TMIS}	Damping matrix of tuned mass inerter system
C_{ae}	Aerodynamic damping matrix
$C_{ae,ti}$	Translational aerodynamic damping
C_L	RMS of lift force coefficient
C_M	Added mass coefficient
d, d_e	Effective diameter of the chimney
d_b	Bottom diameter of the chimney
d_t	Top diameter of the chimney
D	Tip diameter of cylinder
f	Dimensionless F
f_s	Strouhal frequency in Hz
f_{TMIS}	Optimum frequency of tuned mass inerter system
F	The effect of cylinder on the wake

F_L	Lift force developed due to vortex shedding
H	Height of the chimney
$\bar{H}, \hat{h}$	Frequency response function
I_v	Turbulence intensity
k_p	Peak factor
k_s	Stiffness of chimney
k_{TMIS}	Stiffness of tuned mass inerter system
K_{ae,t_i}	Translational aerodynamic damping parameter
$K_{a,t0}$	Translational aerodynamic parameter constant
$\hat{K}$	Ratio of the structural damping to aerodynamic damping
K_1	Dimensionless circular frequency
K_0	Natural dimensionless circular frequency
L	Length of cylinder
$L(\bar{T})$	Probability of response
L_c	Correlation length in diameters.
L_v	Vortex force per unit length
m	Combined mass of structure and fluid
$m_{eq,n}$	Equivalent mass of the chimney
m_{TMIS}	Mass of tuned mass inerter system

$\bar{m}_1$	First modal mass of chimney
M	Dimensionless mass number
M_s, C_s, C_{ae} and K_s	Mass, structural damping, aerodynamic damping, and stiffness matrices
N	Total number of tuned mass dampers placed on chimney
P_f	Probability of failure
r	Length of the segment, or absolute distance between two nodes
R_e	Reynold's number
S_c	Scruton number
S_{FF}	PSDF matrix of vortex lift force
S_t	Strouhal number
S_{yy}	PSDF matrix of response of chimney
t	Time interval
$\bar{T}$	Duration of the wind force acting on the structure
U	Free stream velocity
U_{cr}	Critical velocity
U_m	Mean wind speed
U_r	Reduced velocity
y	Normalized across-wind displacement of chimney
y_{max}	Maximum normalized displacement of chimney

Y	Across-wind displacement of chimney
σ_L	Limiting RMS displacement which is equal to $a_L d$
$\sigma_{Lm}(z)$	Standard deviation of the limiting displacement
σ_y^2	Variance of displacement of chimney
ϕ	Mode shape of chimney
ϕ_1	First mode shape of chimney
Φ	Relative phase difference between the fluid and structural displacement
λ	Ratio of height to diameter of chimney
$\bar{\lambda}$	Optimized coefficient used in Wake oscillator model
ξ_a	Aerodynamic damping
ξ_s	Structural damping ratio
ξ_{TMIS}	optimized damping ratio of tuned mass inerter system
ρ	Density of fluid
ω_1	First structural fundamental frequency in rad/s
ω_s	Strouhal frequency in rad/s
$\bar{\omega}$	Reference angular frequency
δ	Reduced angular frequency
γ	Stall parameter
μ	Mass ratio defined as ratio of damper mass to structural mass

$\bar{\mu}$ Mass ratio dependent of mass number

β_i Inerter ratio

ACRONYMS

ATMD	Active-Tuned Mass Damper
CPSDF	Cross- Power Spectral Density Function
MDOF	Multi Degree of Freedom
MTMD	Multi-Tuned Mass Damper
PSDF	Power Spectral Density Function
SATMD	Semi Active-Tuned Mass Damper
SDOF	Single Degree of Freedom
TLD	Tuned Liquid Damper
TMD	Tuned Mass Damper
TMIS	Tuned Mass Inerter System
VIV	Vortex Induced Vibration