

# ON THE DESIGN OF INFINITE IMPULSE RESPONSE (IIR) DIGITAL NOTCH FILTERS

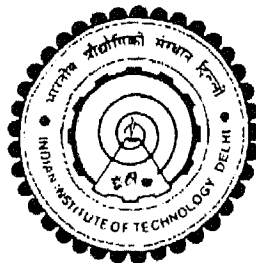
by

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*to the*



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NEW DELHI-110016, INDIA

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*DEDICATED*  
*TO*  
*MY PARENTS*

## CERTIFICATE

This is to certify that the thesis entitled, "ON THE DESIGN OF INFINITE IMPULSE RESPONSE (IIR) DIGITAL NOTCH FILTERS", being submitted by *Yashwant Vasantrao Joshi* to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy** is a record of bonafide research work carried out by him under my supervision and guidance and that in my opinion, it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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Yashwant Vasantry Joshi

## ABSTRACT

The research work presented in this thesis is concerned with new approaches for the design of IIR digital notch filters. These include single frequency notch filters, notch filters with different passband gains, multiple notch filters and two dimensional notch filters with quadrantal symmetry. Allpass based realisations are proposed to make the filters relatively immune to the numerical inaccuracies. Emphasis is given on the development of analytical formulations wherever possible; in other cases, numerical / graphical procedures have been employed.

A method for the design of IIR notch filters based on the first order real allpass filter is first investigated, wherein a new amplitude change function (ACF) has been used. It has been shown that the resulting filter satisfies the given specifications exactly. For very small bandwidths, the usual second order notch filter poles approach zeros and instability problems may arise when the coefficients are quantised to a limited number of bits. By application of this new ACF to such filters, it has been shown that narrow bandwidths can be achieved with *improved* stability as well as passband characteristics.

In certain applications such as ElectroOculography, notch filters with different passband gains are required. A design method for this type of filters has been investigated, based on the transformation of IIR lowpass filters of the Butterworth, Chebyshev, Inverse Chebyshev and Elliptic types. A computational procedure for determining the order  $N$  of the required lowpass filter is given.

Analytical solution has been worked out for the approximations used and the necessary algorithms have been developed.

In the case of multiple notch filters, existing designs satisfy the notch frequency specifications exactly while the 3-dB rejection bandwidth specifications are met approximately and that too, only for very small 3-dB rejection bandwidths. Two different approaches for the design of multiple notch filters have been presented in this thesis. The first one is based on allpass filter while the second one is based on the squared magnitude function. In both the methods, the notch frequency specifications are satisfied exactly while the realised 3-dB rejection bandwidths are always less than those specified.

Two simple transformations have been proposed for the design of two dimensional IIR single and multiple notch filters with quadrantal symmetry. It has been shown that a separable (or superposition of separable) two dimensional sinusoidal interference can be removed from the images affected by such interference.

The proposed design methods have been verified by designing several IIR notch filters. Computer programs have been written and tested in the MATLAB computing environment on a Pentium system using double precision arithmetic.

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