A Thesis

on

VARIATIONAL FORMULATIONS FOR NON-LINEAR DIFFERENTIAL EQUATIONS AND INITIAL VALUE PROBLEMS

by

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SYNOPSIS OF THE PH.D THESIS ON

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The Thesis entitled "Variational formulations for non-linear differential equations and initial value problems" can be envisaged as consisting of two parts. First part, which comprises of five chapters, deals with the problems of existence and formulations of variational principles for non-linear/linear differential equations and Hamilton's principle based on variational principle. Part-II, which has only two chapters is concerned with the development of approximate variational methods that provide solutions to non-linear initial value problems. The chapterwise details of the work put up in this thesis are as follows:

CHAPTER-I

Introduction and Scope of the Subject

The aim of this chapter is two fold in the sense that besides giving a critical review of the related literature we have, for our future reference, re-derived necessary and sufficient conditions for the existence and formulation of variational principle for a system of fourth order non-linear differential equations. This is followed by the deductions of the consistency conditions for the situations dealt in Chapters II-IV.

CHAPTER-II

On the Existence and Formulations of Variational Principles for Single Scalar Nonlinear Differential Equations

Herein, we have utilized the conditions derived in Chapter-I, to study the existence and formulations of potential|alternate potential principles for nonlinear differential equations governing the following physical situations.

(i) generalized Korteweg de-Vries equation,

(ii) two-dimensional irrotational diabatic flow,

(iii) two-dimensional isoenergetic rotational adiabatic flow, and

(iv) large oscillations of a moving thread-line.

CHAPTER-III

On the Existence and Formulations of Variational Principles for Systems of Nonlinear Parital Differential Equations

Unlike Chapter-II, wherein we had confined our attention to study the existence and formulations of variational principles for single scalar nonlinear differential equations, we have in this chapter, applied the consistency conditions to construct functionals for systems of nonlinear differential equations. More specifically, potential|alternate potential principles for the following problems have been formulated:

(i) non-linear waves in a cold plasma,

(ii) two-dimensional irrotational diabatic flow,

(iii) visco-plastic torsion of a prismatic bar of square cross section and

(iv) non-linear buckling of a thin circular plate.

CHAPTER-IV

Variational Principles for Linear-Initial Value Problems

The purpose of this chapter is to develop a procedure to formulate potential|alternate potential principles for linear initial value problems with homogeneous|non-homogeneous initial as well as boundary conditions. The

procedure evolved has been illustrated by means of the following problems:

(i) burning of gas in a rocket,

(ii) isothermal reactor with axial mixing and

(iii) unsteady magneto-hydrodynamical flow.

CHAPTER-V

Hamilton's Principle in Magnetogasdynamics

Using variational principle based on Hamilton's principle for the case of three dimensional unsteady magnetogasdynamical flow we have, in this chapter, attempted to illustrate the importance of Lagrange multipliers with special reference to their existence. The main results of the investigations have been put up in the form of certain Lemmas and Theorems. In order to have an insight to the lemmas derived for the general case we have discussed the case of transverse flow.

CHAPTER-VI

Composite Variational Principles for Nonlinear Initial Value Problems

Keeping in view the fact that the consistency conditions for the existence of a potential|alternate potential principle are quite stringent and severely restrict the class of operators which admit variational formulations we have, in this chapter, utilized the

apparatus of the theory of composite variational principles to deduce the lumped models for the following physical situations:

(i) the transverse deflection of an extensible beam and

(ii) isothermal reactor with axial mixing.

These models are, further, used to find approximate solutions to the respective problems.

CHAPTER-VII

Quasi Variational Methods and Nonlinear Waves

After developing quasi variational principles we have, in this chapter, studied the asymptotic behaviours of nonlinear wave problems governing the following physical situations:

(i) generalized Korteweg de-Vries equation and

(ii) unsteady two-dimensional incompressible meteorological flow equation.

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